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FRACTURE TOUGHNESS EVALUATION FROM INSTRUMENTED SUB-SIZE CHARPY-TYPE TESTS

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ABSTRACT: The possibilities to derive fracture toughness from small specimens are naturally limited due to constraint requirements which are especially restrictive in toughness testing. In this paper two possible ways are explored theoretically and experimentally to use instrumented sub-size Charpy tests to evaluate fracture toughness related data that are essentially size independent (and can be compared with the ones obtained from larger specimens, e.g. standard Charpy specimens). The first way is to adapt a J-R-curve estimation procedure for analyzing instrumented Charpy tests that has proven to deliver conservative fracture toughness data. The second is to scale-up the characteristic values of the instrumented sub-size tests, which are determined to be the total fracture energy and the energy consumed up to maximum load, and then use the evaluation procedure for standard Charpy specimens mentioned above. The corresponding scaling laws are derived analytically, using simplified mechanical models. The results are compared with experimental data obtained from different specimen sizes.

KEYWORDS: Instrumented Charpy test, sub-size specimens, fracture toughness, fracture energy, scaling laws, J-Integral, J-R-curve, CTOA, steel

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In toughness testing often conflicting requirements have to be fulfilled. On one side a minimum degree of constraints is required which leads to a certain minimum specimen size and on the other using small specimens is advantageous in terms of costs, test equipment and material needs. Actually, the specimen size is often limited by the dimensions of the available testing material. That is why testing of small specimens became an important issue in fracture mechanics. A test specimen is called small if it does not fulfill the size requirement of J-testing at least in the upper shelf toughness range. Typical small scale specimens are those of the size of Charpy specimens.

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Since rate-induced hardening contributes to increasing the constraints at a crack or a sharp notch tip and consequently helps to weaken the size requirements, it is advantageous to perform small scale tests under rapid loading. Charpy V-notch impact testing is one of the most widely used test methods to get information on the toughness characteristics of a material (see e.g. [1]). Using an instrumented pendulum hammer and preferably pre-cracked and side grooved specimens, even more sophisticated toughness testing is possible which allows one to evaluate fracture toughness data in terms of J or K_I [2, 3]. A simple single specimen technique to evaluate the force-displacement diagram delivered from the instrumentation has been developed by Schindler [4 - 6]. By means of semi-empirical correlation formulas even the data from instrumented Charpy testing on standard V-notched specimens can be used to estimate fracture toughness values [6 - 8].

However, there are situations where there is not enough testing material available to machine Charpy specimens, e.g. if the broken halves of Charpy specimens are to be used for further toughness tests to characterize the aging behaviour. Such specimens are called sub-sized or miniature Charpy specimens. Because of the reduced constraints and the usually heavily violated size requirement the aim and purpose of such a test is just to obtain toughness-related data that can be compared with those of standard or precracked Charpy tests, rather than to obtain fracture toughness directly.

Basically there are two ways to obtain toughness-related properties from sub-size specimens: The first is to use the same evaluation procedure as for precracked Charpy specimens, [4 - 6], and evaluate fracture toughness-related parameters which are expected to be size independent. The second is to scale up the required experimental input data, which mainly are the total fracture energy and the energy consumed up to maximum force, from the sub-size to the standard Charpy specimens. In the present paper these two ways of evaluating sub-size Charpy tests are explored theoretically and experimentally. First, the above-mentioned fracture toughness evaluation procedure for Charpy type impact testing is recapitulated. Since the access to some of the given references [4 - 6] might be a little difficult, the theoretical derivations are given in concise form in the appendix. Then, based on the same mechanical model, scaling laws for the key parameters are theoretically developed. The results are finally verified and discussed by comparisons with experimental data.

FRACTURE TOUGHNESS EVALUATION FROM IMPACT BENDING TESTS

Theoretical Model and Basic Assumptions

Consider a beam-shaped specimen containing an edge crack or sharp notch of an initial length a_0, loaded in three point bending (Fig. 1). Assuming the fracture mechanism to be ductile tearing (so-called upper shelf behaviour), the force versus displacement diagram obtained from the instrumented test looks as schematically shown in Fig. 2. The fracture process is assumed to consist essentially of two phases (Fig. 3): phase I from the beginning of the loading up to about maximum load which includes crack tip blunting, crack initiation and the first stage of tearing, and phase II as the remaining phase of fracture which is essentially stable tearing. The basic assumption in this model are:

i) phase I is governed by the J-Integral
ii) the tearing crack growth of phase II is governed by the crack tip opening angle (CTOA), which is assumed to be constant during phase II

<u>FIG. 1:</u> Mechanical system of the impact bending test

It can be shown experimentally and theoretically (see appendix) that the crack extension Δa_m at the end of phase I is in general still within the J-controlled region, which is limited to about $0<\Delta a<b_0/10$, according to ASTM Standard Test Method for J_{Ic} (ASTM E813), or ESIS Recommendations for Determining the Fracture Resistance of Ductile Materials (ESIS P2). According to these standards the J-R-curve in phase I is assumed to be of the general form

$$J(Da) = C \cdot Da^{p} \qquad \text{for } \Delta a<\Delta a_m \tag{1}$$

Assumption ii) enables a relation between the fracture energy and CTOA to be obtained, where the latter is proportional to the slope of the J-R-curve at the beginning of the tearing phase II, i.e. at $\Delta a=\Delta a_m$. The corresponding equations are derived in the appendix (eq. (A11-A14)). Since the J-R-curve is a continuous curve three transition conditions have to be fulfilled at the transition point from range I to range II, i.e. where $\Delta a=\Delta a_m$: equality of J, equality of the slope and equality of the curvature of the J-R-curve (see A.15). These conditions enable the three unknowns in (1), C, p and Δa_m, to be determined. One obtains:

$$C = \left(\frac{2}{p}\right)^{p} \cdot \frac{h(a_0)}{B\,(W-a_0)^{1+p}} \cdot W_{tot}^{\;p} \cdot W_{mp}^{\;1-p} \tag{2a}$$

$$p = \left(1+\frac{W_{mp}}{2W_{tot}}\right)^{-1} \tag{2b}$$

$$Da_m = \frac{W_{mp} \cdot p \cdot b_0}{2W_{tot}} \tag{2c}$$

W_{tot} ($=W_{mp}+W_{tm}$; see Fig.2) denotes the total fracture energy and W_{mp} the dissipated **(**non-recoverable**)** part of the absorbed energy at maximum force F. B and $b_0=W-a_0$ are the thickness and width of the ligament, respectively (Fig 1). For side-grooved specimens B shall be replaced by the net thickness B_n. For the present case of three point bending the well known factor ###(a/W) can be approximated by the formula

$$h(a/W) = \begin{cases} 13.818\frac{a}{W} - 25.124\left(\frac{a}{W}\right)^2 & \text{for } 0 < \frac{a}{W} < 0.275 \\ 1.90 + 0.138\frac{a}{W} & \text{for } 0.275 < \frac{a}{W} < 1 \end{cases} \tag{3}$$

which is based on numerical results in [9]. For simplicity the elastic component of J is neglected in (1) and (2), since for small specimens it usually is small compared with the plastic part. However, it can be approximately accounted for by replacing W_{mp} by the total energy at maximum load, W_m.

FIG. 2: Force-vs. displacement diagram

FIG. 3: Schematic representation of a J-R-curve

Note that only W_{mp} (or W_m, respectively) and W_{tot} are required as experimental input data which both can be obtained very easily and unambiguously from the load-displacement diagram (Fig. 2). This feature makes that evaluation procedure particularly easy and suitable for dynamic testing of small scale specimens. The capacity of eqs. (1) and (2) to approximate the J-R-curve has been demonstrated experimentally by comparison with multiple-specimen methods [4, 8].

Near Initiation Fracture Toughness

From the J-R-curve an approximate initiation toughness can be obtained by adapting a procedure given in ASTM E813 or ESIS P2 to determine J_{Ic} or $J_{0.2Bl}$. However, in the case of small or sub-size specimens we recommend a slightly modified, more conservative definition of a characteristic near-initiation toughness, $J_{0.2t}$ as shown and defined in Fig. 3. It represents the J-value at $\Delta a = \Delta a_i + 0.2$mm, where Δa_i is defined as the intersection between (1) and the blunting line. There are several reasons for this modification: First, since $J_{0.2Bl}$ is usually outside the J-controlled range, the J-R-curve for small specimens is often too steep in the region of crack initiation, so the procedures according to ASTM E813 or ESIS P2 lead to a too high $J_{0.2Bl}$; second, it corresponds better to the physical assumption of 0.2

mm tearing crack growth; third, it enables a closed form mathematical expression for $J_{0.2t}$ to be obtained (see below, eq. (5)). For dynamic tests, where the flow stress is increased due to high local strain rates, the slope of the blunting line, s_1, is no longer proportional to the quasistatic flow stress but rather to the dynamic one. Therefore we define

$$J = s_1 \cdot Da = d \cdot s_{fd} \cdot Da \qquad (2 < d < 3) \qquad (4a)$$

where

$$s_{fd} = \frac{F_m \cdot S}{b_0^2 \cdot B} \quad ; \qquad (4b)$$

The factor d in (4a) can be assumed to be about 2. (If the near-initiation toughness is determined by the standard definition of J_{Ic} or $J_{0.2Bl}$ according to ASTM E813 or ESIS P2, then it is recommended to choose the more conservative value 3). Working out the near-initiation toughness $J_{0.2t}$ as defined in Fig. 3, one obtains

$$J_{0.2t} = C \cdot \left[\left(\frac{C}{3 s_{fd}} \right)^{1/(1-p)} + 0.2mm \right]^p + \frac{0.95 \cdot s_{fd}^2 \cdot (W-a)}{E} (1-n^2) \qquad (5)$$

where C and p are given in (2a) and (2b). The second term in (5) represents an approximation of the elastic component of J, which is usually small in the case of small or sub-size specimens and therefore neglected in the J-R-curve given by (1) and (2). As a usually sufficient approximation it can be omitted in (5) as well, if W_{mp} in (2a) is replaced by the full energy at maximum load, W_m.

Sometimes it is convenient to transform fracture toughness values as given in (5) to fracture toughness in terms of the stress intensity factor. The well known transformation formula for this purpose is

$$K_J = \sqrt{J_{0.2t} \cdot E / (1-n^2)} \qquad (6)$$

SCALING LAWS

According to the previous section the key parameters of a load-displacement diagram are the maximum force F_m, the energy at maximum force W_{mp} and the total fracture energy W_{tot}. In the present section scaling laws for these parameters are derived based on the same mechanical models and the same assumptions as used in the previous section. In the following the data corresponding to the sub-size specimens are denoted by a prime, i.e. F_m', W_{mp}' and W_{tot}'. Correspondingly, the geometrical data of the sub-size specimens are also designated by a prime, e.g. W', B', b_0' and a_0'.

Energy at Maximum Force

According to assumption i) the point of maximum force is in general within the J-controlled range of crack extension. This implies that

$$W_{mp} = \frac{J_{mp} \cdot b_0 \cdot B}{\boldsymbol{h}} \tag{7}$$

which is the well known relation derived in [10], and, according to (1),

$$J_{mp} = C \cdot Da_m{}^p \tag{8}$$

C and p are meant to be materials parameters that can be obtained from testing sub-size specimens by using (2a) and (2b), whereas Δa_m as given by (2c) corresponds to a full size test. Inserting these relations into (8) and (7) yields

$$W_{mp} = \frac{B}{B'} \cdot \frac{\boldsymbol{h}'}{\boldsymbol{h}} \cdot \left(\frac{b_0}{b_0{}'} \right)^{1+p} \cdot W'_{mp} \tag{9}$$

where

$$p = \left(1 + \frac{W'_{mp}}{2W'_{tot}} \right)^{-1} \tag{9a}$$

Total Fracture Energy

As can be seen from Fig. 3 the total fracture energy is given by

$$W_{tot} = W_{mp} + W_{tm} \tag{10}$$

The scaling law for the first term is given in eq. (9). The second term is obtained from (A11), where CTOA and σ_{fd} are meant to be material properties that can be determined by a sub-size specimen test by means of (A11) and (4b), respectively. Applying then (A11) to a larger specimen and using a first order approximation, one readily finds

$$W_{tm} = \frac{B}{B'} \cdot \frac{\boldsymbol{h}'}{\boldsymbol{h}} \cdot \left(\frac{b_0}{b_0{}'} \right)^{2} \cdot W'_{tm} \tag{11}$$

or, by inserting (9) and (11) into (10)

$$W_{tot} = \frac{B}{B'} \cdot \frac{\boldsymbol{h}'}{\boldsymbol{h}} \cdot \left[\left(\frac{b_0}{b_0{}'} \right)^{p+1} \cdot W'_{mp} + \left(\frac{b_0}{b_0{}'} \right)^{2} \cdot (W'_{tot} - W'_{mp}) \right] \tag{12}$$

Maximum Force

Since the force F_m is proportional to the moment M and the latter is proportional to Bb^2 (according to (A2)),

$$F_m = \frac{B}{B'} \cdot \frac{S'}{S} \cdot \frac{(b_0 - Da_m)^2}{(b_0' - Da_m')^2} \cdot F_m' = \frac{B}{B'} \cdot \frac{S'}{S} \cdot \frac{b_0^{\,2}\,(1 - {pW_{mp}}/{2W_{tot}})^2}{b_0'^{\,2}\,(1 - {pW_{mp}'}/{2W_{tot}'})^2} \cdot F_m' \qquad (13)$$

and since $pW_{mp}/(2W_{tot}) << 1$ and $W_{mp}/(2W_{tot})$###$W_{mp}'/(2W_{tot}')$, (13) simplifies as a first approximation to

$$F_m = \frac{B}{B'} \cdot \frac{S'}{S} \cdot \frac{b_0^{\,2}}{b_0'^{\,2}} \cdot F_m' \qquad (14)$$

VALIDATION BY EXPERIMENTAL RESULTS

To verify the formulas derived above we consider the experimental data that are obtained in a round robin program on instrumented sub-size Charpy testing performed within the ESIS technical committee 5 [11, 12]. The round robin is concentrated on sub-size specimens according to DIN 50115, but some participants, among them EMPA, also used half-size Charpy specimens. The geometrical parameters of the considered specimens are given in Table 1. The test material was ASTM A533 B Class 1 pressure vessel steel. The values of W_m, W_{tot} and F_m as experimentally determined are given in Table 2.

TABLE 1: *Geometrical parameters of the used specimens.*

	B [mm]	W [mm]	b_0 [mm]	S [mm]	###
standard Charpy	10	10	8	40	1.76
sub-size A	3	4	3	22	1.88
sub-size B	5	5	4	22	1.76

The experimental data given in Table 2 are used to check whether or not the near initiation toughness delivered by eq. (5) or (6) are size independent, and to what degree the scaling laws (9), (12) and (14) are able to predict the corresponding values of the standard size. Only the upper shelf data are included in this consideration, since there is a temperature shift between the different specimen sizes, which means that the values in the brittle-to ductile transition (BDT-) range deviate from one another at a given temperature.

First the key experimental data for the sub-sized specimens as given in Table 2 are scaled up to the size of standard Charpy specimens by using the corresponding formulas. The corresponding data are summarized in Table 3. Regarding the facts that these formulas are purely theoretically derived, without any adjustable factor, and that the (natural) scatter of these parameters is usually of about ###5%, the agreements are generally very satisfying, revealing that the rather coarse analytical models and assumptions as used above to describe the tearing fracture process are adequate.

TABLE 2: *Experimental data measured in the upper shelf range*

	standard Charpy (1)	sub-size A (2)	sub-size B (3)
W_m [J]	65.2	2.26	8.35
W_{tot} [J]	215.2	8.42	29.4
F_m [KN]	19.6	1.32	4.21

(1) provided by SCK.CEN ; (2) mean values of the round robin [8]
(3) mean values of 4 tests performed at EMPA

TABLE 3: *Key parameters of the force-displacement diagram scaled-up to standard Charpy size*

		standard Charpy		sub-size A		sub-size B	
	equation		deviation		deviation		deviation
W_m [J]	(9)	65.2	-	51.1	-21.6%	61.3	-6.0%
W_{tot} [J]	(12)	215.2	-	207.4	-3.62%	229.7	6.7%
F_m [KN]	(14)	19.6	-	17.2	-12.2%	18.5	-5.6%

To obtain size-independent fracture toughness related data two ways are proposed in the introduction: either applying eq. (5) directly to the measured data shown in Table 2, or first scaling-up the basic input data to reference size (which in general will be the standard Charpy size) and then applying (5) to the resulting parameters. Using the first approach results in the values shown in Table 4. The agreement between the parameters for the different sizes is remarkably good. Using the second way results in agreements between the different sizes that is even somewhat better (Tab. 5). From general experience with these kinds of tests it is estimated that the presented deviations are still within the normal scatter band of these parameters. The dynamic flow stress σ_{fd} is necessarily the same for both procedures. Its deviation from one specimen size to the other reflects first of all the different constraint conditions. Thus, as expected, sub-size A specimens exhibit the lowest dynamic flow stress.

TABLE 4: *Near-initiation toughness calculated by the given equations directly from the measured data given in Table 2*

		standard Charpy		sub-size A		sub-size B	
	equ.		deviation		deviation		deviation
$J_{0.2t}$ [N/mm]	(5)	339	-	286	-15.6%	331	-2.3%
K_J [$N/mm^{1.5}$]	(6)	8845	-	8124	-8.2%	8739	-1.2%
σ_{fd} [MPa]	(4b)	1225	-	1075	-12.2%	1157	-5.5%

TABLE 5: *Near-initiation toughness calculated from the scaled-up data given in Table 3 and the geometrical parameters of standard Charpy specimens*

		standard Charpy		sub-size A		sub-size B	
	equ.		deviation		deviation		deviation
$J_{0.2t}$ [N/mm]	(5)	339	-	302	-10.9%	345	1.8%
K_J[$N/mm^{1.5}$]	(6)	8845	-	8348	-5.6%	8922	0.87%
σ_{fd} [MPa]	(4b)	1225	-	1075	-12.2%	1157	-5.5%

EXTENSIONS

Scaling Fracture Energy From Non-Instrumented Tests

Due to the lack of W_{mp} the scaling law for W_{tot} (eq. (12)) can not be applied when performing non-instrumented tests. However, since W_{mp} is in general much smaller than W_{tot} (usually $W_{mp} < W_{tot}/3$) p is expected to have a numerical value close to 1 according to (2b). Therefore, a first order approximation of (12) is

$$W_{tot} = \frac{B}{B'} \cdot \frac{h'}{h} \cdot \left(\frac{b_0}{b_0'}\right)^2 \cdot W_{tot}' \tag{15}$$

Scaling Laws in the Brittle to Ductile Transition Regime

Since the presented equations are based on the assumption of a purely ductile tearing process, the J-R-curve represented by (1) and (2) holds only in the upper shelf regime. However, as discussed in [2], the near initiation toughness defined in (5) represents a lower bound value in the transition regime and the lower shelf range. Thus (5) or (6) can be applied in the entire temperature range, to obtain conservative BDT- fracture toughness versus temperature curves. As an example, Fig 4 shows the BDT- curves obtained for the material considered in the previous chapter using standard and sub-size specimens. The scatter of the data of each specimen type is relatively large. Nevertheless, the expected temperature shift between the different specimen types is clearly visible. Note that the upper shelf values of the three specimens are in good agreement with each other.

FIG. 4: *Fracture toughness measured using different specimens as a function of temperature*

J-R-Curves for Standard Size Specimens

It has been recognized before [4] that the J-R-curves described by (1) and (2) are in general too flat (i.e. conservative), because the numerical value of the exponent p as given in (2b) is somewhat too high compared with experimental data. Correspondingly the resulting fracture toughness values as given by (5) are conservative, which usually is a rather welcome feature in small specimen testing, since the latter tend to exhibit too high fracture toughness values due to the lack of sufficient constraints. However, if for certain reasons best estimate J-R-curves for pre-cracked Charpy specimens are desired, then p as given in (2b) should be modified to

$$p = \frac{3}{4} \cdot \left(1 + \frac{W_{mp}}{W_{tot}}\right)^{-1} \qquad (16)$$

This semi-empirical modification is based on comparison with multiple-specimen J-R-curves [4]. Applying p as given in (16) directly to the sub-sized specimen data leads to J-R-curves that differ significantly from each other. Thus, if best estimate J-R-curves are required, then the second way according to the preceding chapter should be chosen, which means using the scaled-up energy values in eq. (1), (2a) and (2b). For the examples considered in the previous chapter this results in the J-R-curves that are shown in Fig. 5. Therein the η-Factor was tentatively reduced from 1.76 (according to (3)) to 1.46 as suggested in [7] in order to account for the finite notch radius. However, the effect of the sharpness of the notch root on the J-R-curve, which is also considered in [5] and [8], needs further clarification.

FIG. 5: *"Best estimate" J-R-curves obtained from specimens of different sizes*

CONCLUSIONS

The direct evaluation of standard fracture toughness data from sub-size Charpy specimens is certainly a too ambitious aim, since it is not possible to fulfill size and constraint requirements. Furthermore, there often is no fatigue crack introduced in sub-size specimens for practical reasons. A more realistic aim is to determine characteristic parameters that are comparable with the ones obtained from standard size Charpy tests. For this purpose scaling laws for the main characteristic values, i.e. the energy at maximum load and the total fracture energy, have been derived analytically based on simple mechanical models. The agreement with experimental results showed, that the corresponding simple mechanical models are adequate to describe the tearing fracture process.

As a suitable parameter to characterize the toughness of small and sub-size specimens the near-initiation toughness $J_{0.2t}$ is proposed. This parameter is closely related to J_{Ic} and has proven to be able to characterize the J-value at about initiation of ductile tearing of precracked standard-size Charpy specimens. The present investigation reveals that this value is essentially size-independent which means that the upper shelf value is about the same for different specimen sizes. Concerning size-independence the best results seem to be achievable when the experimental data that are required in the determination formula of $J_{0.2t}$ are first scaled-up to the standard Charpy size by the proposed scaling laws.

The proposed toughness property $J_{0.2t}$ is able to characterize the toughness behaviour in the brittle-to ductile transition range. Since the required experimental data are well defined and easily obtained, it is reproducible and applicable throughout the entire toughness range**.** For these reasons this parameter is well suited for determining the temperature shift in brittle-to-ductile transition, e.g. due to specimen size, impact speed or irradiation damage.

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APPENDIX

Basic Relations in the Tearing Phase

The tearing energy consumed in phase II (see Fig. 3) is determined by

$$W_{tear}(\Delta a) = \int_{\theta(\Delta a_m)}^{\theta(\Delta a)} M(\Delta a) \cdot d\theta = \int_{b_0-\Delta a_m}^{b_0-\Delta a} M(b) \cdot d\theta(b) \quad , \qquad \text{(A1)}$$

with $b = b_0 - \Delta a$ denoting the actual ligament width and M the bending moment. The latter can be written in terms of a representative flow stress σ_{fd} as

$$M(b) = \frac{c}{4} \sigma_{fd} \cdot B \cdot b^2 \quad , \qquad \text{(A2)}$$

where c represents a non-dimensional factor that takes the value 1 for plane stress and about 1.45 for plane strain.

FIG A.1: *Definition of the parameters at the fracturing section of an edge cracked plate in bending*

Assuming the crack tip opening angle CTOA, which is defined as

$$CTOA = \frac{d\boldsymbol{d}}{da} \tag{A3}$$

(with δ denoting the crack tip opening displacement) to be constant in phase II leads to the kinematical relation

$$d\boldsymbol{q}(b) = -\frac{CTOA}{z \cdot b} db \tag{A4}$$

where zb denotes the distance between the crack tip and the center of rotation. Inserting these relations into (1) gives

$$W_{tear}(Da) = \frac{B \cdot c}{8z} CTOA \cdot \boldsymbol{s}_{fd} \cdot [2b_0 \cdot (Da - Da_m) - (Da - Da_m)^2] \tag{A5}$$

According to [9] the formal J-R-curve in range II is represented by

$$J(Da) = \frac{\boldsymbol{h} \cdot W_{mp}}{B \cdot b_0} + \frac{\boldsymbol{h} \cdot W_{tear}(Da)}{B \cdot (b_0 - Da)} \qquad \text{for } \Delta a > \Delta a_m \tag{A6}$$

By inserting (A5) in (A6), then taking the derivative with respect to Δa and comparing the resulting expression at $\Delta a = \Delta a_m$ with the derivative of the well known fundamental equation

$$J = \boldsymbol{s}_{fd} \cdot m \cdot \boldsymbol{d}(Da) \tag{A7}$$

and (A3) one obtains

$$\frac{dJ}{da} = \frac{dJ}{dDa} = \frac{\boldsymbol{h} \cdot c}{4z} CTOA \cdot \boldsymbol{s}_{fd} = \frac{d\boldsymbol{d}}{da} \cdot m \cdot \boldsymbol{s}_{fd} \quad , \tag{A8}$$

which leads, with using (2), to

$$z = \frac{\boldsymbol{h} \cdot c}{4m} \tag{A9}$$

The nondimensional factor m as introduced in (A7) depends on the triaxiality of the local loading state and is of the order of 1.
Obviously (see Fig. 2)

$$W_{tear}(\Delta a=b_0-\Delta a_m) = W_{tm} \tag{A10}$$

so (A5) and (A9) result in

$$CTOA = \frac{2\eta \cdot W_{tm}}{B \cdot m \cdot \sigma_{fd} \cdot (b_0 - \Delta a_m)^2} \tag{A11}$$

<u>Determination of J-R-Curve</u>

From (A6), (A11) and (1) it follows that the J-R-curve is formally given by

$$J(\Delta a) = C \cdot \Delta a^p \qquad \text{for } \Delta a\text{###}\ \Delta a_m \tag{A12a}$$

$$J(\Delta a) = J_{mp} + s_2 \left[(\Delta a - \Delta a_m) - \frac{(\Delta a - \Delta a_m)^2}{2b_0} \right] \qquad \text{for } \Delta a > \Delta a_m \tag{A12b}$$

where

$$J_{mp} = \frac{\eta \cdot E_{mp}}{B \cdot b_0} \tag{A13}$$

$$s_2 = \frac{2 \cdot \eta \cdot (W_{tot} - W_{mp})}{B \cdot (b_0 - \Delta a_m)^2} \tag{A14}$$

The three unknowns Δa_m, C and p are determined by the following matching conditions of eqs. (A12a) and (A12b) at $\Delta a=\Delta a_m$:

$$J^I(\Delta a_m) = J^I(\Delta a_m) = J_{mp} \tag{A15a}$$

$$\frac{dJ^I}{d\Delta a}(\Delta a_m) = \frac{dJ^{II}}{d\Delta a}(\Delta a_m) = s_2 \tag{A15b}$$

$$\frac{d^2 J^I}{d\Delta a^2}(\Delta a_m) = \frac{d^2 J^{II}}{d\Delta a^2}(\Delta a_m) = -\frac{s_2}{b_0} \tag{A15c}$$

Herein, the superscripts I and II indicate correspondence to range I (i.e. eq. (A12a)) and II (A12b), respectively (see Fig. 3). One obtains therefrom

$$C = \left(\frac{2}{p}\right)^{p} \cdot \frac{\boldsymbol{h}(a_0)}{B\left(W - a_0\right)^{1+p}} \cdot W_{tot}{}^{p} \cdot W_{mp}{}^{1-p} \qquad \text{(A16)}$$

$$Da_m = \frac{W_{mp} \cdot p \cdot b_0}{2W_{tot}} \qquad \text{(A17)}$$

$$p = \left(1 + \frac{W_{mp}}{2 \cdot W_{tot}}\right)^{-1} \qquad \text{(A18)}$$

$W_{tm} = W_{tot} - W_{mp}$

W_{tm} W_{tm} W_{tm}
$W_{tot} - W_{mp}$

$E_{tm} = E_{tot} - E_{mp}$

$E_{tm} =$
$E_{tot} - E_{mp}$

$J_{0.2t}$ $J_{0.2t}$
$J_{0.2t}$

J_m
J_m

W_{tm} W_{tm} W_{tm} W_{mp} W_{tm}

J_m $J_{0.2t}$ J_{mp}

$W_{tot} - W_{mp}$

$E_{tm} = E_{tot} - E_{mp}$

$E_{tm} =$
$E_{tot} - E_{mp}$

$J_{0.2t}$ $J_{0.2t}$
$J_{0.2t}$

J_m
J_m

(see [1]):

$$Da_m = \frac{W_{mp} \cdot p \cdot b_0}{2W_{tot}}$$