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## Determination of Transferable Lower-Bound Fracture Toughness from Small Specimens

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**ABSTRACT:** The master-curve (MC) enables fracture toughness to be measured in the ductile-to-brittle temperature (DBT) regime of ferritic steels using small specimens. However, MC application to structural components remains unclear in some areas. Vital issues of regulatory concern are the differences in the reference temperature  $T_0$  obtained from specimens of different sizes and shapes, the extension of the MC into the upper transition range, and the acceptability, or not, of a given level of failure probability to be used in the analysis of safety-relevant components [e.g., nuclear power reactor pressure vessel (RPV)]. Based on general theoretical considerations, the existence of a deterministic temperature-dependent lower bound of  $K_{Ic}$  or  $K_{Jc}$  is postulated. It can serve to provide conservative fracture toughness values as required in a screening analysis, which is usually the first step of a fracture mechanics analysis of a structural component. A simple method to determine this lower bound from the reference temperature  $T_0$  is presented. Furthermore, since testing of small specimens, like in precracked Charpy (PCC) tests, often leads to E1921-invalid data, an alternative procedure to estimate the MC is suggested that can use the invalid data. The practicability of the method is shown by a few examples. Based on the same method, a lower-bound of dynamic fracture toughness can be estimated from Charpy data sets. Combined with an empirical temperature shift, the lower bound of static fracture toughness can be estimated as well.

**KEYWORDS:** fracture toughness, transferability, lower-bound, validity, brittle-to ductile transition, master-curve

### Introduction

In engineering fracture mechanics the overall loading state of a crack in a structural component is quantified by parameters like the stress intensity factor (SIF)  $K_I$  or  $J$  integral. The crack's behavior in a component can be predicted by comparing the calculated applied SIF or  $J$  integral with the critical ones ( $K_{Ic}$  or  $J_{Ic}$ ) measured on a material specimen. A key issue of this concept is the transferability of the corresponding loading parameters from the laboratory specimens to the engineering structures, since they may depend on the size and shape of the specimen or component, respectively. For the sake of conservatism, restrictive size criteria are required for the small specimens to qualify the critical  $K_I$  as a transferable  $K_{Ic}$  value in linear-elastic fracture mechanics (LEFM). In elastic-plastic fracture mechanics (EPFM), these size requirements are less severe, but additional requirements concerning the local crack-tip constraints must be satisfied. Usually, an engineering failure assessment is performed step by step, starting with a simple LEFM analysis using conservative assumptions concerning crack size and plane strain fracture toughness data. Only if the safety cannot be clearly demonstrated by such a conservative approach, more complex models have to be applied, including EPFM, with consideration of the specific conditions concerning constraints, residual stresses, strain rates, etc. However, experience has shown that more complex models and sophisticated numerical methods do not necessarily lead to more accurate predictions of the fracture behavior because the required additional parameters are often not available with sufficient accuracy. Simple conservative approaches also serve well to check the plausibility of the results delivered by more complex and less transparent ones. A key element of such a screening analysis is the availability of conservative fracture toughness data. This paper deals with the question of how to obtain a corresponding lower-bound curve as a function of temperature from a limited number of tests on small specimens.

In the ductile-to-brittle transition (DBT) range of ferritic structural steels, the inherently high scatter in fracture toughness makes a statistical evaluation of the test data necessary. For ferritic steels, an evaluation

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procedure based on Weibull statistics and a master curve (MC) that is characterized by just one material parameter, the reference temperature  $T_0$ , is given in ASTM E 1921 [1,2]. However, relatively little is known about the extrapolation of the MC to low failure probabilities as appropriate in a practical engineering safety analysis. There is also some doubt for principal theoretical reasons to what extent a one-parameter-approach such as the MC can cover the cleavage process adequately, since the latter depends—as reviewed later in this paper—on at least two independent material parameters. Actually, brittle fracture of a high-risk-structure such as the reactor vessel of a nuclear power plant is an event that must be excluded under any circumstances—not only by a certain low probability. For these reasons, a deterministic lower-bound fracture toughness curve  $K_{Jc}(T)$  that holds in a conservative way for any possible crack in an arbitrarily large structural component without further restrictions would be preferable. The main questions are, whether or not such a deterministic lower bound exists and how to determine it experimentally. In practical approaches such as the ASME code [3] a similar lower-bound fracture toughness curve  $K_{Jc}(T)$  is postulated based on numerous experimental data, and an empirical relation between  $T_0$  and the nil-ductility reference temperature  $RT_{NDT}$  is given. However, from a regulatory point of view, a safety analysis of a vital component based on an essentially empirical approach may be considered as insufficient. Theoretical support for such relations is required as well.

As discussed below by theoretical considerations, a temperature-dependent deterministic lower bound of fracture toughness of ductile metals can be assumed to exist even in the DBT range. The question of how to determine it from a limited number of small specimens in a traceable, conservative way is the main one. In the present paper an attempt is made to determine it based on well established empirical and simple theoretical relations. The resulting lower-bound curve as a function of the temperature depends on the reference temperature  $T_0$  and some additional material properties. Furthermore, a simplified procedure is proposed that enables the estimation of  $T_0$  from data sets that contain too many E 1921-invalid data, which often is the case when small specimens such as precracked Charpy (PCC) specimens are tested at relatively high temperatures. As shown by an example, this procedure can be applied analogously to determine lower-bound fracture toughness from standard Charpy data sets.

### Validity versus Transferability

Validity according to a certain testing standard is in general not equivalent to an unconditioned transferability, i.e., the direct use of the corresponding material data in a fracture analysis. In classical fracture mechanics, transferability is accounted for by using E399-valid plane-strain fracture toughness data [4], i.e., values that fulfill the size criteria of LEFM,

$$B, a, W, \dots \geq \alpha \cdot \left( \frac{K_{Jc}}{R_p} \right)^2, \quad (1)$$

where  $R_p$ ,  $B$ ,  $a$ , and  $W$  denote the yield stress, thickness, crack, length, and in-plane width, respectively. By criterion 1 it is made sure that the size of the plastic zone is small compared with the in-plane as well as the out-of plane dimension of the specimen. This guarantees that plane strain behavior and full in-plane constraints dominate in the small plastic volume at the crack tip, and that the crack's stability is then governed by a Griffith-type energy criterion. Accordingly, if Eq 1 is fulfilled, then  $K_{Jc}$  is transferable without further restrictions.

If analysis methods according to EPFM are applied, then validity according to ASTM E 1820 requires

$$B, a, W, \dots \geq \alpha_{EPFM} \cdot \frac{J_{Ic}}{\sigma_f}, \quad (2)$$

where  $\sigma_f$  is the mean value of the yield stress  $R_p$  and the ultimate tensile strength  $R_m$ . The constant  $\alpha_{EPFM}$  is 25 in the case of ductile crack extension, or 100 in case of cleavage fracture [5], respectively. Criterion 2 makes sure that the fracture process zone is located within the  $J$ -controlled region. To determine the MC in the DBT-regime,  $\alpha_{EPFM}=30$  is required, with the plastic flow stress  $\sigma_f$  being replaced by  $R_p$  in Eq 2. To compare these requirements, Eq 2 is turned into the form of Eq 1 by using the basic relation  $J=K_7^2(1-\nu^2)/E$ . Evaluated for the properties of a typical structural steel ( $R_p=500$  MPa,  $E=210$  GPa, and  $R_p/\sigma_f=0.8$ ) the following numbers for  $\alpha$  are obtained:

$$\text{LEFM, ASTM E 399: } \alpha = 2.5, \quad (3a)$$

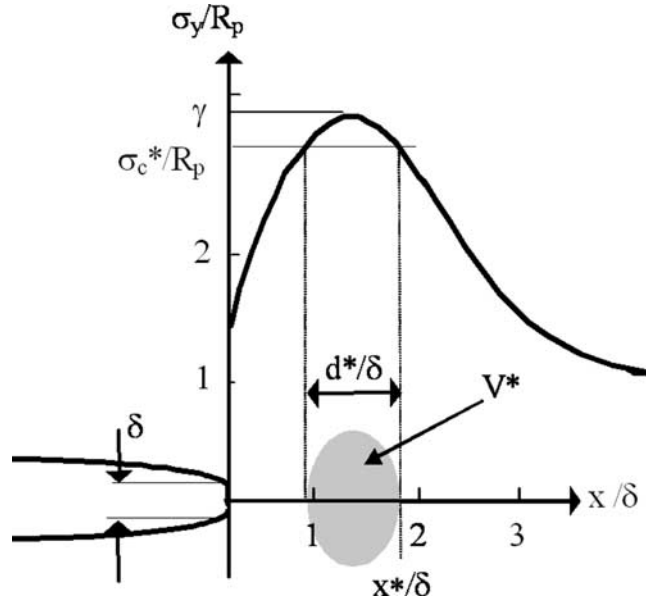


FIG. 1—Nondimensional representation of the stress distribution in the vicinity of a crack-tip.

$$\text{EPFM, ASTM E 1820, ductile} \quad \alpha = \frac{25 \cdot R_p \cdot (1 - \nu^2)}{E} \cdot \frac{R_p}{\sigma_f} = 0.043, \quad (3b)$$

$$\text{EPFM, ASTM E 1820, cleavage} \quad \alpha = \frac{100 \cdot R_p \cdot (1 - \nu^2)}{E} \cdot \frac{R_p}{\sigma_f} = 0.17, \quad (3c)$$

$$\text{EPFM, ASTM E 1921, MC} \quad \alpha = \frac{30 \cdot R_p \cdot (1 - \nu^2)}{E} = 0.065. \quad (3d)$$

As it can be seen from Eqs 3, the size requirements for MC evaluation are considerably less restrictive than those of  $J_{Ic}$  determination according to ASTM E 1820. This is explicable by the fact that the MC is related to the behavior of a 1T-CT-specimen as a reference, i.e., not to a full constraint big specimen or component. Thus, E 1921-validity does not necessarily imply unconditioned transferability of the MC to components. The statistical size correction that is part of the ASTM E 1921 procedure only enables  $T_0$  to be determined from specimens of different sizes, but does not account fully for the total size effect when it is applied in a failure assessment of a large component.

According to Refs. 3 and 6 the MC can be transferred to components through its empirical relation to the ASME lower-bound  $K_{Ic}$ -curve, which is given by

$$K_{Ic} = 36.5 + 22.8 \cdot \exp[0.036 \cdot (T - RT_{NDT})], \quad (4)$$

with

$$RT_{NDT} = T_0 + 19.4 \text{ } ^\circ\text{C}. \quad (5)$$

However, a purely empirical basis is not quite satisfying when dealing with events of potentially catastrophic consequences, like a brittle fracture of a nuclear reactor pressure vessel (RPV). Theoretical support based on physical models is required as well. The theoretical aspect of brittle fracture in the DBT region is considered in the next section.

### Physics of Cleavage in the DBT-Regime

Occurrence of cleavage fracture in an elastic-plastic material requires at least two criteria to be fulfilled [7]: the stress in the vicinity of the crack,  $\sigma_{y\max}$ , has to exceed a certain minimum value  $\sigma_c^*$ , and the region where it fulfills this condition must cover a certain minimum area in front of the crack tip, characterized by  $x^*$  in Fig. 1. The peak stress is given by the constraint parameter  $\gamma$  as defined in Fig. 1. The second

criterion may be interpreted as an energy criterion [8]. Thus, the conditions for cleavage crack initiation can be formulated as follows:

- (i) The maximum stress  $\sigma_{y\max} = \gamma \cdot R_p$  in the vicinity of the crack tip (see Fig. 1) must exceed the cleavage stress  $\sigma_c^*$ , i.e.,

$$\gamma \cdot R_p > \sigma_c^*,$$

where  $R_p$  denotes the tensile yield stress and  $\gamma$  a constraint parameter in the range  $1 < \gamma < 3.5$ , depending on the constraint condition.

- (ii) The elastic energy  $W_{el}^* = \int_{V^*} U_{el} dV$  stored in a certain volume  $V^*$  of the width  $d^*$  in the  $x$  direction (Fig. 1) must be sufficient to produce a cleavage fracture on the area  $0 < x < x^*$  ( $U_{el}$  denoting the elastic strain energy per unit volume).

If these two criteria are fulfilled over a distance  $z \gg x^*$  along the crack front, a cleavage fracture can spontaneously spread out in the  $x$  and  $z$  directions. However, these two conditions being fulfilled does not mean that an unstable cleavage fracture is necessarily initiated. It needs to be triggered by a brittle particle or weak bond in the critical zone just ahead of the crack front. The probability of the presence of such a potential cleavage trigger in the relevant zone increases with increasing crack size and load level. This adds the probabilistic component to the problem of cleavage.

According to criterion (i), a cleavage fracture is principally possible for temperatures below  $T_{DBT}$ , where the latter is given by the equation

$$R_p(T = T_{DBT}) = \sigma_c^* / \gamma. \quad (6)$$

Since the cleavage stress  $\sigma_c^*$  is not very temperature dependent [9,10],  $T_{DBT}$  can be determined, in principle, from Eq 6, if  $\gamma$  and  $R_p(T)$  are known. For  $T > T_{DBT}$  cleavage fracture is excluded.

Using the basic relation

$$J = m \cdot R_p \cdot \delta \quad (7)$$

and the proportionalities

$$U_{el} \propto (\gamma \cdot R_p)^2; \quad V^* \propto \delta^2, \quad d^* \propto \delta; \quad x^* \propto \delta, \quad (8)$$

one finds from criterion (ii) and Eq 6 the following general relation between  $J_c$ ,  $R_p(T)$ , and the constraint parameters  $\gamma$  and  $m$  (see Appendix for the derivation)

$$\frac{J_c(T) \cdot R_p(T) \cdot \gamma^2}{m} = \text{const.} \quad \text{for } \gamma > \sigma_c^* / R_p. \quad (9)$$

The ratio  $\gamma^2/m$  depends on the crack-tip constraints [8] and varies in the range of 1 for plane stress and about 6 for plane strain with saturated in-plane constraints, i.e.,

$$1 < \gamma^2/m < (\gamma^2/m)_{\text{sat}} \approx 6. \quad (10)$$

According to Eq 9, the main reason for the temperature dependence of  $J_c$  is the temperature dependence of the yield stress. Eq 9 enables the effect of constraints and temperature on fracture toughness to be quantified.

In a hypothetical perfectly homogeneous elastic-plastic material a step-wise transition from cleavage to ductile tearing would occur at  $T = T_{DBT}$  as sketched schematically in Fig. 2. However, in a real polycrystalline material, such as RPV steel, the transition starts at somewhat lower temperatures—due to many physical reasons such as grain boundary plasticity—and cleavage is possible due to work-hardening even after some amount of tearing, i.e. somewhat above  $T_{DBT}$  (shaded area above initiation). Furthermore, in Fig. 2 the effect of a small variation of the peak stress  $\gamma \cdot R_p$  is shown. The shaded area between the thick and the thin lines represents the resulting variability of  $J_c$ . At temperatures near  $T_{DBT}$ , even a small change of  $\gamma \cdot R_p$  causes a big change of fracture toughness  $J_c$ . There are many reasons for a certain variability of  $\gamma \cdot R_p$ , including loss of local constraints due to blunting or internal void growth, loss of global constraints due to plasticity or tearing crack extension, irregular (instead of perfectly straight) crack front, free surfaces, and local geometry of the crack front. Positive feedback, such as loss of constraints due to

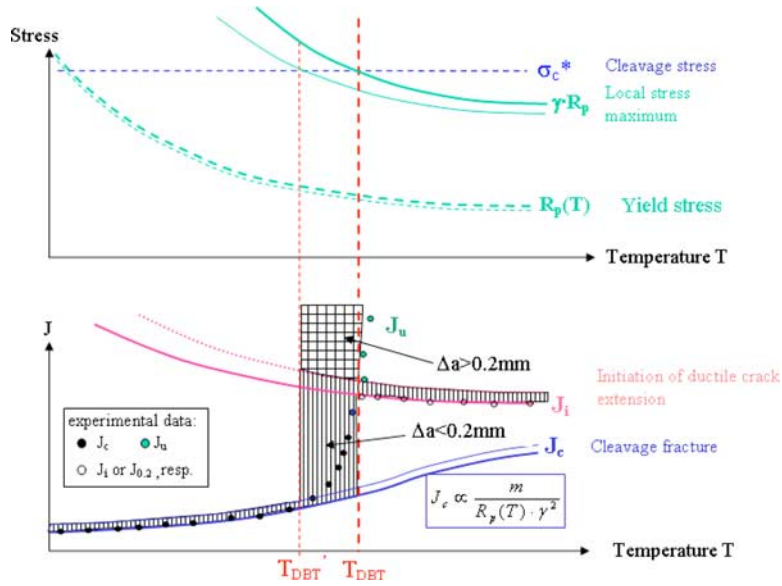


FIG. 2—Schematic representation of the critical  $J$  integral as a function of temperature, and the scatter band corresponding to a variability of  $(\gamma R_p)$  of a few percent.

plasticity and ductile tearing, amplifies these effects. Accordingly, a high scatter of  $J_c(T)$  is to be expected in the DBT regime even for “deterministic” reasons. The additional requirement of the presence of a weak particle as a trigger only adds to the stochastic nature of cleavage.

Since the conditions (i) and (ii) need to be fulfilled under any circumstance, regardless whether or not a brittle particle to act as a trigger is present, the equations derived therefrom, i.e., Eqs 6 and 9, represent lower bounds of cleavage fracture toughness. According to Eqs 6 and 9 it is dependent on the temperature and the constraints. If the latter are “saturated,” i.e., the ratio  $\gamma^2/m$  takes its maximum possible value  $(\gamma^2/m)_{\text{sat}}$ , then the corresponding curve represents an absolute, conservative lower bound of  $J_{Ic}(T)$  or  $K_{Jc}(T)$  for the considered material. The question of how to determine this curve is dealt with in the following.

### Determination of Lower-Bound $K_{Jc}(T)$

Consider a material that is characterized by the reference-temperature  $T_0$  according to ASTM E 1921 [1]. By using the empirical relation 5 one finds that the MC matches the ASME lower-bound  $K_{Ic}$  curve 4 in a conservative way for a nominal probability of failure (p.o.f.) of 1 %, which is described by the equation

$$K_{Jc} = 23.5 + 24.4 \cdot \exp[0.019 \cdot (T - T_0)]. \quad (11)$$

Thus, although associated with a nominal p.o.f. of 1%, Eq 11 can be considered as an empirical lower bound of the critical SIF of a 1T-CT-specimen within the validity range of the MC, i.e., for  $T_0 - 50^\circ\text{C} < T < T_0 + 50^\circ\text{C}$ .<sup>3</sup> As long as  $K_{Jc}$  fulfills condition 1, which is

$$K_{Jc} \leq \sqrt{\frac{B}{\alpha}} \cdot R_p, \quad (12)$$

(with  $\alpha=2.5$  according to ASTM E 399 and  $B=0.0246$  m according to ASTM E 1921), the  $K_{Jc}$  values can be regarded as valid in the sense of LEFM, thus transferable without any further restriction. Correspondingly, Eq 11 represents the lower bound of fracture toughness for temperatures  $T < T_{\text{LEFM}}$ , where the latter is defined as the temperature at the intersection of the LEFM-limit 12 as a function of temperature and the 1 %-MC, thus,

<sup>3</sup>Some new experimental data obtained in a current research project from relatively large specimens imply that Eq 11 may not be conservative in some cases. Once these new data are fully evaluated, a revision of Eq 11 is eventually to be considered.

$$T_{\text{LEFM}} = T_0 + \frac{1}{0.019} \cdot \ln \left[ \frac{0.0992 \cdot R_p(T_{\text{LEFM}}) - 23.5}{24.4} \right]. \quad (13)$$

This equation can be solved for  $T_{\text{LEFM}}$  easily by one or two iterations.

According to Merkle et al. [11,12], plane strain fracture toughness  $K_{Ic}$  can be estimated from  $K_{Jc}$  in the DBT range analytically by the following equations:

$$K_{Ic} = (M_1^{1/3} + M_2^{1/3})^2 \cdot K_{\min}, \quad (14)$$

where

$$M_1 = G + \sqrt{G^2 - (1/3)^3}, \quad (15)$$

$$M_2 = G - \sqrt{G^2 - (1/3)^3}, \quad (16)$$

$$G = \frac{1}{2} \left( \frac{K_{Jc}}{K_{\min}} - 1 \right) \left( \frac{R_p \cdot \sqrt{B/\alpha}}{K_{\min}} \right), \quad (17)$$

with

$$B = 0.0246 \text{ m}, \\ \alpha = 2.5, \quad K_{\min} = 20 \text{ MPa} \cdot \text{m}^{0.5}. \quad (18)$$

Application of Eqs 14–18 on Eq 11 results in a lower-bound curve  $K_{Ic\text{-LB}}(T)$  for the temperature range  $T_{\text{LEFM}} < T < T_0 + 50^\circ\text{C}$ .

To extrapolate  $K_{Ic\text{-LB}}(T)$  to temperatures  $T > T_0 + 50^\circ\text{C}$ , where Eq 11 no longer holds, the general relation 9 can be used. It delivers

$$K_{Jc}(T) = K_{Jc}(T_0 + 50^\circ\text{C}) \cdot \sqrt{\frac{R_p(T_0 + 50^\circ\text{C})}{R_p(T)}} \quad \text{for } T_0 + 50^\circ\text{C} < T < T_{\text{US-max}}. \quad (19)$$

The upper limit of the validity range of Eq 19,  $T_{\text{US-max}}$ , represents the upper boundary of the DBT range and is defined as the maximum value of  $T_{\text{DBT}}$  according to Eq 6, i.e.,  $T_{\text{DBT}}$  for saturated constraints. Physically, it is meant to be the temperature above which no cleavage fracture is possible even after a stable crack extension  $\Delta a$  within the  $J$ -controlled regime. Its experimental determination from small specimens is difficult. For the sake of simplicity we suggest to use the ASME lower bound curve, which includes experimental data from large specimens, in the form 4 and 5. Correspondingly,  $T_{\text{US-max}}$  can be interpreted as the temperature where Eq 4 reaches the value of  $K_{JQ}(T)$  at a stable crack extension within the  $J$ -controlled region, which is about  $\Delta a < 2.5$  mm for a 1T-CT-specimen. According to experimental results from the literature and own data (see Fig. 3 as an example), the critical SIF  $K_{Iq}$  at a crack extension of  $\Delta a = 2.5$  mm can be as high as about twice the one at initiation ( $\Delta a = 0.2$  mm). By these assumptions one obtains

$$T_{\text{US-max}} = T_0 + \frac{1}{0.036} \cdot \ln \left( \frac{2K_{Ji} - 36.5}{11.3} \right). \quad (20)$$

For  $T > T_{\text{US-max}}$ , i.e. in the upper shelf, the transferable part  $K_{Jc\text{-LB}}$  of  $K_{Jq}$  is given by the validity limit 2 with  $\alpha_{\text{EPFM}} = 25$ , thus,

$$K_{Jc\text{-LB}} = \sqrt{\frac{E \cdot \sigma_f(T_{\text{US-max}}) \cdot b_0}{25(1 - \nu^2)}}, \quad (21)$$

where  $b_0$  denotes the ligament size of the small test specimens.

The lower-bound fracture toughness estimated from small specimens by the procedure explained above is shown in Fig. 4 (fat dotted line) as a function of  $T - T_0$  for a material with an assumed yield-stress of  $R_p(T) = (600 - T + T_0)$  MPa. It consists of Eq 11 for  $T < T_{\text{LEFM}}$ , Eqs 14–18 applied on Eq 11 for  $T_{\text{LEFM}} < T < T_0 + 50^\circ\text{C}$ , Eq 19 for  $T_0 + 50^\circ\text{C} < T < T_{\text{US-max}}$ , and Eq 21 for  $T > T_{\text{US-max}}$ . In Fig. 5, this lower bound is evaluated from the data of PCC specimens of a A508 class 2 steel reported in Ref. [11] and compared with the data from larger specimens of the same material. To determine  $T_{\text{BDT-max}}$ , it was assumed that the

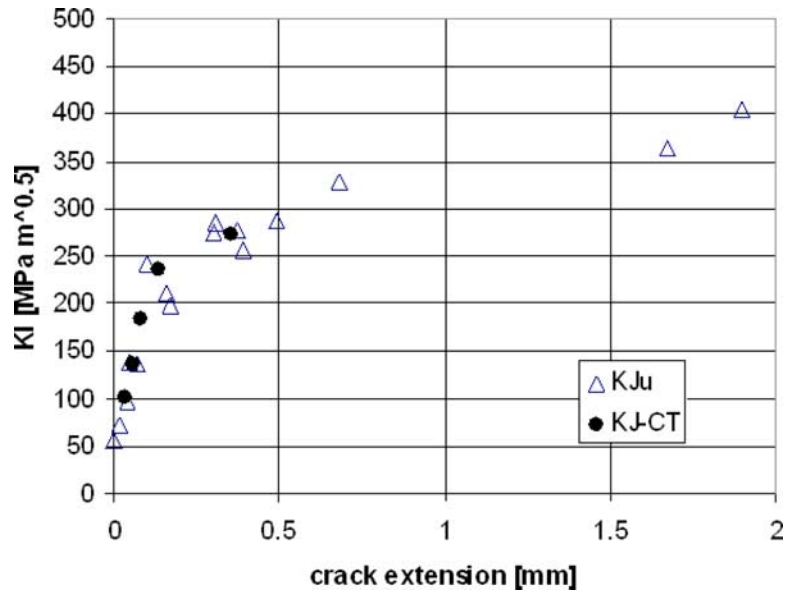


FIG. 3—Critical SIF  $K_{Ju}$  at onset of cleavage as a function of corresponding crack extension  $\Delta a$  for steel ST355 for precracked Charpy and 1T-CT specimens.

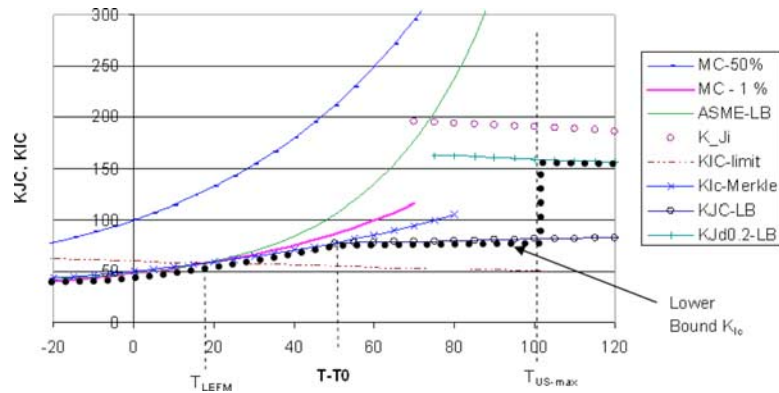


FIG. 4—Lower bound fracture toughness (fat dotted line) in relation to the 1 %-failure probability MC and the ASME lower-bound curve with  $T_{NDT} = T_0 + 19,4^\circ\text{C}$  [for  $R_p(T_{LEFM}) = 580\text{ MPa}$ ].

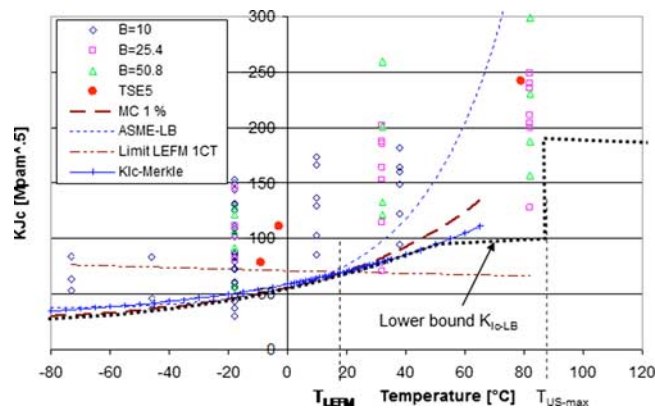


FIG. 5—Lower-bound fracture toughness determined from precracked Charpy specimens ( $B = 10$ ) in comparison with data from 1''- and 2''-CT-specimens ( $B = 24.4$  and  $B = 50.8$ ) and surface cracks under thermal shock loading (TSE5) (experimental data from Ref. [11]).

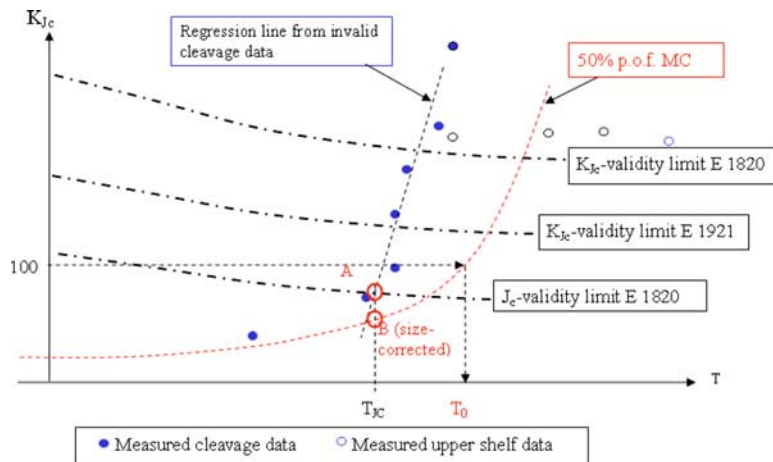


FIG. 6—Estimation procedure for  $T_0$  from an insufficient data set of small specimens test.

upper shelf toughness in terms of  $K_{Jc}$  is about  $250 \text{ MPa} \cdot \text{m}^{0.5}$ . One can see from Fig. 5 that the lower bound envelopes nearly all the data, including those from the thermal shock experiment on a full-sized vessel in the upper transition region (TSE5 according to Refs. [11,12]). The few outliers are explicable by the conservative way these rather old data were evaluated.<sup>4</sup>

### Estimation of $T_0$ from Invalid PCC Tests

If small specimens are used and testing is performed at temperatures above  $T_0$ , then some of the measured  $K_{Jc}$  values may exceed the validity limits, which can lead to a E 1921-invalid  $T_0$ . Anyway, if  $T_0$  is used just to estimate a lower bound fracture toughness as suggested in the previous section, ASTM E 1921-invalid approximations may be sufficient in a conservative engineering analysis based on lower bounds. In the following an evaluation procedure that includes invalid data of small specimens is suggested.

Experimental DBT curves measured on small specimens exhibit a significantly lower  $T_{DBT}$  than larger ones, whereas the behavior at about  $T_{LEFM}$  in the lower transition range is more or less independent of specimen size. Therefore, the apparent drop of  $K_{Jc}$  in the DBT regime of small specimens is steeper than the one of larger specimens.<sup>5</sup> The steeper the drop of  $K_{Jc}$ , the less sensitive is  $T_0$  with respect to data scatter. In case of small specimens, when only an insufficient number of valid test data are available, one can take credit from this behavior to relax the standard requirements of ASTM E 1921 adequately. A correspondingly modified procedure to determine  $T_0$  is sketched in Fig. 6. It works as follows: The  $K_{Jc}$  values from specimens broken by cleavage in the range  $\Delta a < b_0/10$ , but not fulfilling criterion 2 with  $\alpha_{EPFM} = 100$ , are extrapolated by linear regression down to the validity limit given by Eq 2 with  $\alpha_{EPFM} = 100$  (Fig. 6). The point of intersection (A) represents a valid  $K_{Jc}$  point in the sense of ASTM E 1820 and corresponds to a p.o.f. of 50 %. The size correction according to ASTM E 1921 leads to point B, which represents  $K_{Jc}$  at the same temperature and p.o.f. for a 1T-CT-specimen. Obviously, this point is part of the 50 % -p.o.f.-MC, so  $T_0$  is determined.

By this procedure  $T_0$  can be estimated from a data set that is insufficient according to ASTM E 1921. An example is shown in Fig. 7. The experimental data were obtained from tests performed on an instrumented pendulum hammer according to ISO 14556, and evaluated by the single-specimen method described in Refs. [13–15]. There were not enough valid cleavage data to determine  $T_0$  by the standard procedure, so the estimation procedure explained above is applied. From the resulting  $T_0$ , the lower-bound fracture toughness curve was determined according to the previous section. As expected, the apparent DBT of the PCC specimens occurs at a much lower temperature than the DBT estimated for saturated constraints, and the corresponding drop is relatively steep, so the modified determination of the reference temperature  $T_0$  explained above cannot lead to significant errors.

<sup>4</sup>According to private communication with one of the authors of Ref. [11].

<sup>5</sup>In the case of impact loading, this behavior is enhanced due to adiabatic effects: Plastic yielding in the ligament results in local heating, which causes a reduction of the yield stress and the constraints.

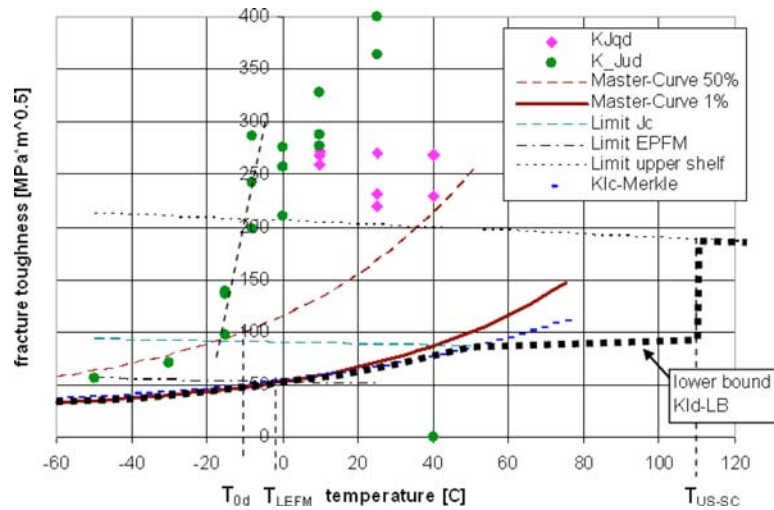


FIG. 7—Lower bound dynamic fracture toughness for Steel St 355 determined according to Eq 15 from precracked Charpy specimens under impact loading.

### Estimation of Lower-Bound Fracture Toughness from Charpy Tests

The estimation procedure described in the previous section can be extended to estimate lower bounds of fracture toughness from Charpy data sets in a similar way. An example of an unirradiated pressure vessel steel is shown in Fig. 8. The data points denoted by  $K_{JCVN}$  are calculated by the semianalytical correlation formula

$$K_{JCV} = \sqrt{\frac{5.20 \cdot A_g \cdot KV \cdot E}{\left(1 - 1.47 \cdot \frac{KV}{R_m}\right) \cdot (1 - \nu^2)}} \quad (22)$$

from the Charpy fracture energy KV, where  $R_m$  denotes the engineering tensile strength and  $A_g$  the uniform engineering fracture strain of the material. Eq 22 corresponds to the analytical relation between  $J_{Ic}$  and KV as derived in Ref. [15], lowered by a constraint correction factor of 0.71 [8,13]. This formula holds for upper shelf and upper transition region. By the procedure explained in the previous section, the reference temperature  $T_{0CVN}$  (i.e.,  $T_0$  for the case of a Charpy test) is determined by a linear extrapolation of the data lying above the validity limit according to Eqs 1 and 3c to the corresponding limit curve, as shown in Fig. 6. The size correction according to ASTM E 1921 applied to the point of intersection A leads to point B.

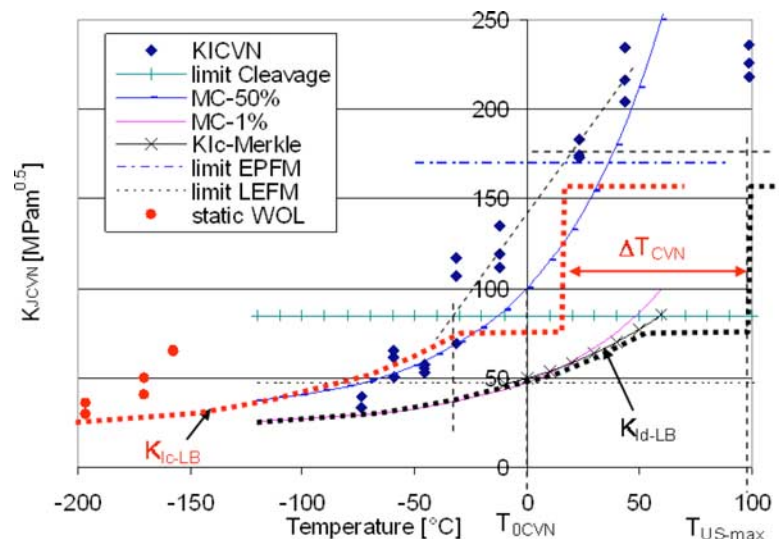


FIG. 8—Estimated lower bounds of dynamic and static fracture toughness for reactor pressure vessel steel from Charpy data.

The condition that  $B$  is located on the 50 % p.o.f MC, leads to  $T_{0\text{CVN}} = -1^\circ\text{C}$ . Therefrom, the lower-bound dynamic fracture toughness  $K_{Ic-LB}$  can be determined by the procedure described above.

Note that  $T_{0\text{CVN}}$  and the corresponding lower bound curve correspond to the impact loading rate of a CVN test. According to Ref. [16] there is an empirical relation between the standard static  $T_0$  and  $T_{0\text{CVN}}$  of

$$T_0[^\circ\text{C}] \cong 0.534 \cdot T_{0\text{CVN}}[^\circ\text{C}] - 76.18. \quad (23)$$

In the present case, Eq 23 predicts a temperature shift of  $\Delta T_{\text{CVN}} = T_{0\text{CVN}} - T_0 = 83^\circ\text{C}$ . Thus, the lower bound for static loading,  $K_{Ic-LB}(T)$ , is obtained by shifting the curve  $K_{Ic-LB}(T)$  by the amount of  $-83^\circ\text{C}$ . As shown in Fig. 8, the static fracture toughness values determined on full-sized WOL specimens are located above this boundary, confirming the present approach.

## Conclusions

The transferability of the MC to components is not guaranteed, since the corresponding fracture toughness values do not meet necessarily the size requirement of ASTM E 1820 for brittle fracture, and the underlying Weibull statistics appear to be inadequate to predict the fracture toughness associated with the very low values of failure probability that are required in a safety analysis of vital components. In a component larger than a certain limiting size, the statistical size effect becomes negligible compared with the effect of constraints and yield stress. Therefore, it is justified to postulate a deterministic lower bound of fracture toughness, which holds throughout the transition regime and upper shelf, and which is traceable to conservative criteria of transferability. It is meant to represent fracture toughness values that can be used for a first screening of a structure within a safety analysis or a defect assessment.

As shown in the present paper, such a lower bound  $K_{Ic}$  curve can be obtained by combining the MC with the well-established size criteria according to ASTM E 1820. In the upper transition regime, which is covered neither by the MC nor the upper-shelf fracture toughness, the lower bound curve is obtained by a conservative extrapolation based on a simple theoretical relation between  $K_{Ic}$  and the yield stress.

In case of small specimens, estimation the lower-bound fracture toughness can be obtained with often sufficient accuracy from data sets that do not fulfill the requirements of ASTM E 1921 regarding the number of valid tests. A suitable estimation procedure is suggested in the present paper. By the same method and an analytical correlation formula to determine  $K_{Ic}$  values from Charpy fracture energy, estimates of the lower-bound dynamic fracture toughness can be made from standard Charpy tests. By using an empirical temperature shift, lower-bound static fracture toughness can be estimated.

The suggested procedures are based on simple theoretical models and meant to be for a conservative engineering use. They need further experimental confirmation.

## Appendix: Derivation of Eq 9

Criterion (ii) states that a cleavage fracture requires

$$\frac{W_{\text{el}}^*}{x^*} = \text{const}. \quad (A1)$$

The physical background of the hypothesis underlying criterion (ii) is the following: The extension of a small brittle particle (small compared to  $d^*$ ) to a small crack is governed by a Griffith-type criterion. However, because of the large stress gradients, the corresponding crack can arrest before it triggered a macroscopic cleavage crack. The latter requires that the former event releases enough elastic energy to let the crack propagate all the way back to the macroscopic crack tip. So the total elastic energy  $W_{\text{el}}^*$  stored in the region  $V^*$  (defined as the region, where the stresses exceed the cleavage stress) is what matters most. For the sake of simplicity  $V^*$  can be assumed to be of cylindrical shape of radius  $d^*/2$ . As any distance of the local stress field,  $d^*$  scales with CTOD  $\delta$ , or with  $\delta_c$ , respectively, at onset of cleavage, which occurs at

$$J = J_c. \quad (A2)$$

Thus, the elastic energy (per unit thickness) stored in  $V^*$  is proportional to

$$W_{\text{el}}^* \propto U_{\text{el}} \cdot \delta_c^2, \quad (A3)$$

where  $\delta_c$  is related to  $J_c$  by Eq 7, i.e.,

$$J_c = m \cdot R_p \cdot \delta_c. \quad (\text{A4})$$

The elastic strain energy density is proportional to the square of the local peak stress  $\gamma \cdot R_p$ ,

$$U_{el} \propto (\gamma \cdot R_p)^2. \quad (\text{A5})$$

Inserting Eqs A3–A5 in Eq A1 and using  $x^* \propto \delta$  leads to

$$\frac{J_c(T) \cdot R_p(T) \cdot \gamma^2}{m} = \text{const.},$$

which is Eq 9.

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