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PREFERABLE INITIAL CRACK LENGTH FOR FRACTURE TOUGHNESS EVALUATION USING SUB-SIZED BEND SPECIMENS

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Abstract: In order to guarantee maximum crack-tip constraints, fracture toughness testing standards require an initial crack length of about a_0=W/2. When performing impact tests on specimens smaller than the required size the situation is somewhat different: Since the thickness is usually not sufficient to provide plane strain conditions, it is questionable whether the initial crack length should still be about a_0=W/2 to provide maximum in plane constraints. On impact testing of small specimens, shorter cracks exhibit several experimental advantages, the main ones being, first, the lower influence of dynamic oscillations and second, the extended validity range of the measured toughness properties. For these reasons, we suggest to use shorter cracks and account for the insufficient in-plane constraints by correcting the measured (apparent) fracture toughness values. For this purpose a the theoretical correction formula is derived in this paper. Based on this correction and the standard size requirements one obtains a lower-bound fracture toughness that can be use as a conservative design value. The latter is shown to be maximum for a relative crack length of about a_0/W=0.25 – 0.3.

Keywords: Impact testing, initial crack length, sub-sized, fracture toughness, pre-cracked Charpy specimen, transferability, fracture toughness, constraint.

Nomenclature

a	crack length
A	Non-dimensional constant
A_0	initial cross section of uniaxial tensile specimen
A_f	fracture cross section at failure of a uniaxial tensile specimen
a_0	initial crack length
B	specimen thickness
b_0	initial ligament width, b_0=W-a_0
DBTT	brittle-to-ductile transition temperature
F	Force

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J	J integral
$J_{0.2Bl}$	J integral at onset of stable tearing
J_u	J integral at onset of unstable cleavage
K_{Ic}	SIF at initiation of ductile tearing
m	constraint factor
M	bending moment
R_m	ultimate tensile strength
R_p	yield stress
SIF	stress intensity factor
T	T-stress (constant term of near-tip stressfield)
W	specimen width
z	nondimensional distance of the center of rotation from the crack tip
Z	Reduction of area in the uniaxial tensile test
Δa	stable tearing crack extension
β	maximum T normalized by R_p
δ	crack-tip opening displacement
γ	Nondimensionalized maximum stress in the vicinity of the crack-tip
γ_{mT}	Constraint factor as defined in this paper
σ_{fd}	flow stress σ_f at increased strain rate (dynamic flow stress)

Subscripts:

eff	effective, as measured
fs	full-sized
LB	lower bound
max	maximum
ss	sub-sized

In practical application of fracture mechanics there often are situations where using specimens smaller than those required by the current fracture toughness testing standards like ASTM E 1820 or ISO/DIS 12135 is suitable or even the only possibility. The term "sub-sized" is used here for specimens that do not fulfil the size requirements for a "valid" fracture toughness. This means that the resulting fracture toughness values are not transferable to larger structures without restrictions. However, for purposes like qualitative comparison of different types of materials or monitoring aging effects they still can be useful.

Often impact loading of sub-sized specimens is advantageous, since the increased loading rate and the correspondingly increased local strain rates tend to rise the plastic flow stress and, correspondingly, to extend the size criteria. A typical sub-sized specimen suitable for many purposes is the pre-cracked Charpy specimen (Fig. 1) [1, 2]. Its main advantage is that it can be easily tested dynamically at various temperatures by means of a standard instrumented Charpy pendulum hammer. A simple method to evaluate J are given and discussed in [3, 4]. However, a generally accepted testing and evaluation standard does not exist yet. For this reason, the Swiss regulatory board published a special guideline [6]. Another guideline for testing pre-cracked Charpy specimens is presently in preparation by the European Structural Integrity Society (ESIS), Technical Committee TC5.

Lacking special standards for sub-sized specimen testing and evaluation, the question arises to which degree the requirements of the above mentioned full-size static testing standards have to be adopted, especially concerning the size requirements and the validity criteria. The size requirements ensure that the constraints at the crack-tip are essentially "plane strain". Furthermore, these standards include requirements concerning crack lengths: In edge cracked bending and CT- specimens, for example, the crack-length shall be such that a crack-depths-to-width ratio of about $a_0/W=0.5$ results. This condition shall guarantee maximum in-plane constraints, which are known to affect the measured ("effective") fracture toughness as well. However, when using small specimens, shorter initial cracks exhibit several advantageous features concerning range of validity and transferability, and general experimental behaviour particularly in the case of dynamic testing, as discussed in the next section. From a theoretical point of view one can argue that in sub-sized specimens the out-of-plane constraints are not sufficient for plane strain conditions to prevail anyway. Lacking out-of-plane constraints cannot be fully compensated by higher in-plane constraints. Thus, sub-sized specimens do not lead directly to transferable fracture toughness; corrections for constraints are necessary anyway.

For these reasons the choice of the most favourable initial crack length is a matter of optimising several aspects and depends on the purpose of testing. For example, the Swiss Federal Nuclear Safety Inspectorate (HSK) decided to use specimens with $a_0/W = 0.3$ within the Swiss surveillance program of nuclear power plants [7, 8]. In the present paper, the question of the most beneficial initial crack-length is dealt with regarding maximum validity and transferability of the evaluated fracture toughness values. As far as the theoretical analysis is concerned, a simplified theory to account for constraints suggested previously by the author [9] is applied, which enables experimental data obtained from specimens with reduced constraints to be corrected for maximum constraints. However, qualitatively, the same results could be obtained on purely empirical grounds.

Advantages of shallow cracks

Dealing with small specimens under impact loading, like typically pre-cracked Charpy specimens, shallow cracks[1] exhibit some advantageous features compared with standard deep cracks, including the following:

Maximum Force. When testing small specimens, the maximum force reaches in general the plastic limit load, which is given by

$$F_{\max} = \frac{c_1 \cdot \sigma_f \cdot B \cdot b_0^2}{S} \qquad (1)$$

where $\sigma_f = (R_p+R_m)/2$ represents the flow stress, with R_p and R_m denoting the yield stress (e.g. $R_{p0.2}$) and the ultimate tensile strength, respectively, and c_1 a factor close to 1

[1] By the term "shallow" we denote a crack which is considerably shorter than the standard $a_0/W=0.5$, but still long enough to prevent slip-lines from the crack-tip to the rear surface, which means that $a_0/W > 0.2$.

accounting for the plastic constraints in bending (c_1=1.33 according to ASTM E 1820). Thus F_{max} is increasing with increasing b_0, which makes the force measurement more accurate. Particularly this is true in impact testing (see below).

Elastic Energy Release Rate. The elastic component of J, J_{el}, represents the energy release rate. A relatively high elastic portion of J is advantageous in fracture toughness testing in any case: The higher the elastic component of a given value of J, the more likely a significant amount of unstable cleavage fracture occurs, since J_{el} promotes unstable spreading of cleavage fracture.

It is well known from linear-elastic fracture mechanics that a good approximation of J_{el} for an edge crack a_0>W/3 under pure bending is

$$J_{el} = \frac{K_I^2(F = F_{\max})}{E} \cong \frac{0.92 \cdot F_{\max}^2}{E \cdot B \cdot b_0^3} = \frac{0.92 \cdot (c_1 \cdot \sigma_f)^2 \cdot B \cdot b_0}{S^2 \cdot E} \qquad (2)$$

The third equation follows from (1). Thus J_{el} is increasing with increasing b_0.

Range of Validity. For specimens like pre-cracked Charpy the range of validity of the evaluated fracture toughness values in terms of J is extended for shorter cracks. According to (2) the lower bound fracture toughness value is increased, which is beneficial in any case. Furthermore, the range of validity of J-R-curves evaluated from such tests is increased, since according to [2] the fracture process can be regarded as sufficiently J-controlled for $\Delta a<b_0/10$.

Geometrical Aspects. The shorter a_0, the closer is the shape of the ligament to the square shape of the ligament of a standard CT-specimen as is used in quasi-static testing. The deflection to span ratio at a given J-value is smaller, so the system is less affected by geometrical non-linearity.

Dynamic Effects. Dynamic oscillations of the force signal affect the accuracy of the common quasi-static evaluation. The amplitude of the oscillations is dependent first of all on the impact speed, but not on the crack-length. As shown by (1), the magnitude of the maximum load increases with shorter cracks, so the force signal is less affected by the disturbing oscillations in the case of shorter cracks.

Moreover the loading rate in terms of dJ/dt and the local strain rate at the crack-tip at a constant impact rate are higher.

Validity and Transferability of Fracture Toughness from Sub-Sized Specimens

In principle, sub-sized 3PB specimens (Fig. 1) can be evaluated in the same way as full sized specimens [3]. If the fracture behaviour is completely ductile, then the fracture toughness is characterised by a near initiation J-value denoted by $J_{0.2Bl.}$ If an unstable cleavage fracture takes place, then the characteristic fracture toughness parameter is denoted by J_u (using the terminology of ISO/DIS 12135). According to ASTM E1820 these properties are “valid”, (i.e. transferable to an analysis of cracks in a larger structural part) if the specimen size meets the criteria[2]

$$J_{0.2Bl} \leq Min(\sigma_f \cdot b_0/20,\ \sigma_f \cdot B/20,\ \sigma_f \cdot a_0/20) \quad (3a)$$

$$J_u \leq Min(\sigma_f \cdot b_0/50,\ \sigma_f \cdot B/50,\ \sigma_f \cdot a_0/50) \quad (3b)$$

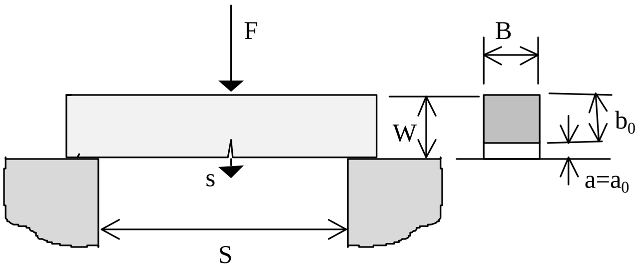

Fig. 1: *Mechanical system of an impact test in three-point-bending (3PB)*

For a specimen under bending with a_0>0.2, the crack load is governed by the ligament width b_0, whereas the crack-length is responsible only for providing the required crack-tip constraints. Thus, the third term in the brackets of eqs (3) can be disregarded in the context of the present paper, where the constraints are accounted for in a different way, as shown below. If we restrict our considerations to specimens with $B>b_0$ and $a_0>0.2W$ (like typically pre-cracked Charpy specimens), then the decisive requirements corresponding to (3) are

$$J_{0.2Bl} \leq \sigma_f \cdot b_0/20 \quad (4a)$$

$$J_u \leq \sigma_f \cdot b_0/50 \quad (4b)$$

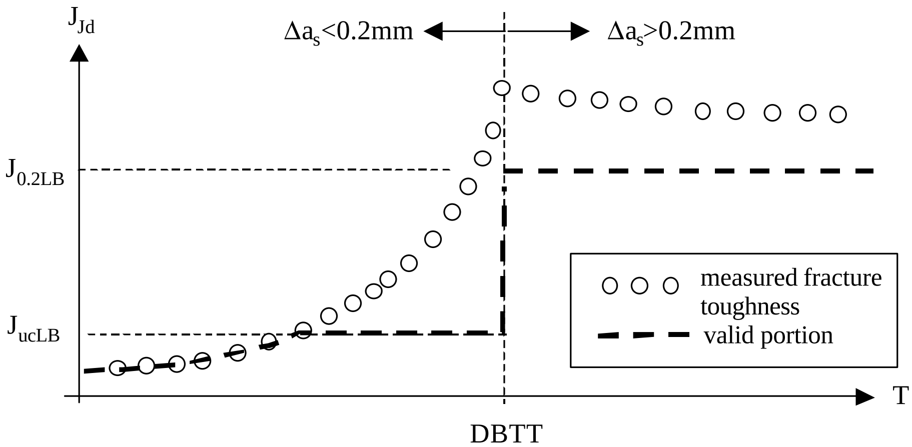

Fig. 2: Fracture toughness as a function of temperature measured by sub-sized specimens, and its valid portion.

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[2] For simplicity we use the numerical value 50 in the denominator of eqs. (3b), (4b), (5b) and (15b) instead of m as defined in ASTM E1820.

If the requirements (4) are violated, then the measured fracture toughness value is “not valid” and the specimen is called “sub-sized”. Nevertheless, testing sub-sized specimen can still be useful. In the first stage of the loading process of the specimen, before the above validity limits are exceeded, the crack is loaded by J-values that fulfil the criteria (4) without showing unstable growth or substantial stable extension. Thus, the highest J-value that fulfils criteria (4a) or (4b), respectively, represents a lower bound of the fracture toughness. Correspondingly, the transferable lower bounds of $J_{0.2Bl}$ or J_u, respectively, are given by

$$J_{0.2Bl,LB} = s_f \times b_0/20, \qquad \text{for } T>DBTT \qquad (5a)$$

$$J_{u,LB} = s_f \times b_0/50 \qquad \text{for } T<DBTT \qquad (5b)$$

DBTT denotes the ductile-to-brittle transition temperature and is defined here as the temperature where the stable crack growth preceding unstable cleavage is Δa_s=0.2 mm. The transferable portion of the measured fracture toughness curve is schematically shown in Fig. 2. As can be seen from (5), the lower bound fracture toughness in the lower shelf as well as in the upper shelf is increased if $b_0=W-a_0$ is increased, i.e. if the initial crack length a_0 is decreased.

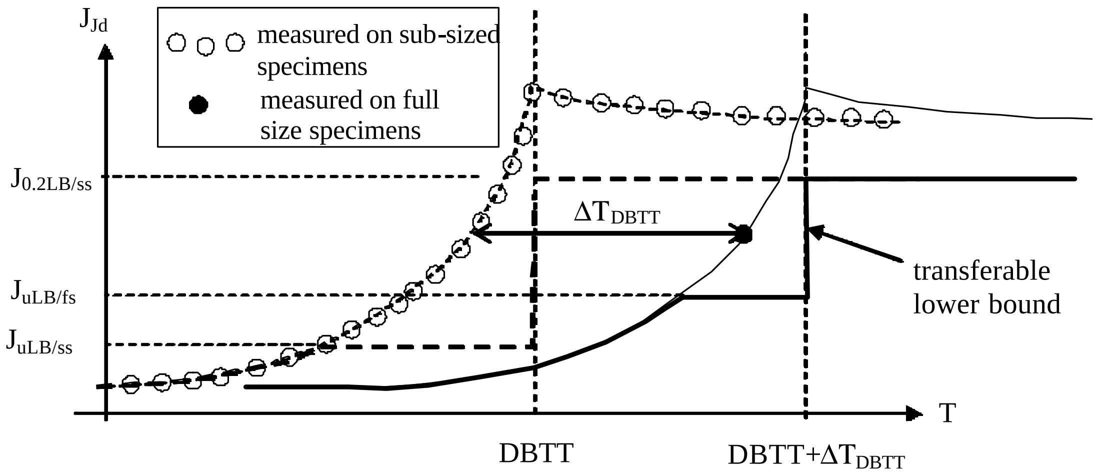


Fig. 3: *Transferable lower bound of fracture toughness based on a test series of sub-sized specimens and a single full-sized test, as a function of temperature.* $J_{ucLB/fs}$ *denotes the lower bound according to (5b) for the full-sized specimen,* $J_{ucLB/ss}$ *the corresponding value for sub-sized specimens*

If the material exhibits a transition behaviour, there is necessarily a shift ΔT_{BDTT} of DBTT due to size, shape and crack-length, which is important to be accounted for when transferring data obtained from sub-sized specimens to larger structures. If ΔT_{BDTT} is not known from experience, it can be determined by a single test in the DBTT-range using a full-size specimen loaded at the desired loading rate (Fig. 3). The transferable portion of the fracture toughness from a series of sub-sized specimen and a single full-size specimen is schematically shown in Fig. 3. Note that the lower bound of the upper shelf toughness corresponds to the sub-sized specimen, whereas the one of the lower shelf corresponds to the full-size specimen. Thus, it is advantageous to use sub-sized specimens that are designed such that the lower bound toughness is maximum. As shown below, this can be achieved by using shallow cracks and an analytical formula to correct the effective lower bound for the missing in-plane constraints.

Effect of Constraint on Fracture Toughness

In order to obtain an analytical expression of the effect of constraints on fracture toughness, the concept as suggested by the author in [9] is applied[3]. Within this concept the constraint is characterized by the non-dimensional peak stress γ defined as (see Fig. 4)

$$\gamma=\sigma_{ymax}/R_p \tag{6}$$

The effect of γ on the fracture toughness was found in [9] to be given by the relations

$$J_u \cdot \frac{g^2}{m} = const \qquad \text{for T < DBTT} \tag{7}$$

$$\frac{J_{0.2Bl}}{m \cdot \left\{ \exp\left[\frac{s_f \cdot Z}{R_p \cdot g \cdot (1-Z)} \right] - 1 \right\}} = const \qquad \text{for T > DBTT} \tag{8}$$

where

$$m = J/ R_p \cdot d \tag{9a}$$
$$Z=(A_0-A_f)/A_0 \tag{9b}$$

Z denotes the standard reduction of area in a uniaxial tensile test, with A_0 and A_f being the cross section initially and after fracture, respectively.

According to [9] the non-dimensional peak stress γ can be estimated by

$$g \cong \frac{g_{mT} \cdot s_f}{R_p} \tag{10}$$

where γ_{mT} denotes the non-dimensional peak stress corresponding to γ for a non-hardening material. According to [9] it can be approximated by

$$\gamma_{mT} \;@\; c_m \cdot m + c_b \cdot b \tag{11}$$

[3] If the reader has no access to Ref. [9], then the analytical derivations of this section, which lead to the constraint correction curves shown in Fig. 6 (a), are hard to understand. If this is the case, the author suggests to consider these curves as empirically obtained by fitting the experimental data shown in Fig 6 (b) and the numerous further data available in the literature. The main conclusions of this paper remain the same.

The factors c_m and c_b are given roughly by:

for plane strain:	$c_m \cong 2$	$c_b \cong 1$	(12a)
for plane stress	$c_m \cong s_f / R_p$	$c_b \cong 0$	(12b)

and

$$\beta = T_{max} / R_p \qquad (13)$$

quantifies the normalized maximum T-stress in the considered specimen, which means the value of the second term of Williams' expansion of the local stress-field at the maximum external load, which corresponds either to the crack-instability at $K_I = K_{Ic}$ or to the plastic collapse, respectively.

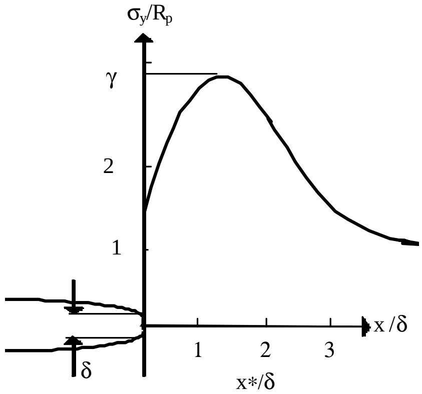


Fig. 4: *Non-dimensional representation of the stress distribution in the vicinity of a crack-tip (schematic)*

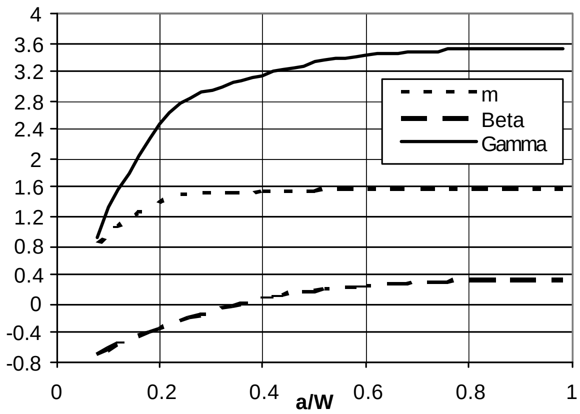


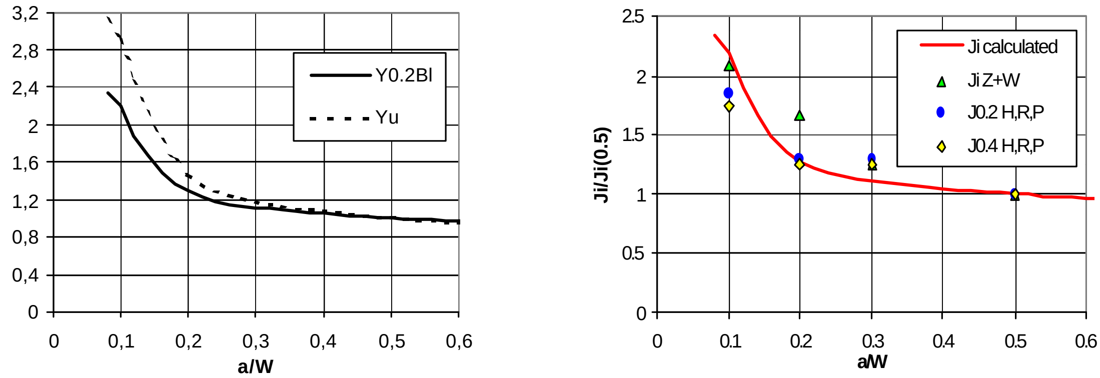
Fig. 5: *Constraint factors m, b (Beta) and g_{mT} (Gamma) of an edge crack in a beam in bending as a function of non-dimensional crack length for non-hardening materials.*

Effect of Crack Length on Constraint of a 3PB-Specimen

To apply (7) and (8) to the considered case of an edge cracked beam under bending, γ has to be quantified by means of (10) and (11). Thus, m(a/W) and β(a/W) have to be known for this crack configuration. β(a/W) is given in [10] and shown in Fig. 5. As derived in the appendix, m can be estimated analytically for full plastic yielding. The behaviour of m for non-hardening materials as a function of non-dimensional crack-length a/W is also shown in Fig. 5, as well as the constraint parameter γ_{mT} resulting from (11) and (12). Using these values in (7) and (8), one obtains the relation between the standard fracture toughness properties J_u and $J_{0.2Bl}$ and the measured fracture toughness on a specimen containing a shallow initial crack, $J_{0.2Bl,eff}$ and $J_{u,eff}$. The corresponding ratios Y_u and $Y_{0.2Bl}$ are defined as

$$Y_u(a/W) = \frac{J_{u,eff}(a/W)}{J_u} \quad ; \qquad Y_{0.2Bl}(a/W) = \frac{J_{0.2Bl,eff}(a/W)}{J_{0.2Bl}} \qquad \textbf{(14)}$$

respectively. The ratios Y_u and $Y_{0.2Bl}$ are shown in Fig. 6 as a function of crack length (for a typical value of Z for a medium-strength structural steel, Z=0.6). They correspond well with experimental data reported in the literature (e.g. [11, 12, 13]).



(a) (b)

Fig. 6, (a): *Effect of the crack length of a 3PB-specimen on the apparent ("effective") fracture toughness for Z=0.6 (see eq. (14) for definition of Y_u and $Y_{0.2Bl}$); (b): validation of (a) by some experimental data from the literature (right) (Z+W from [12], H,R,P from [11]).*

Lower Bound Fracture Toughness

By (14) and using Y_u(a/W) or $Y_{0.2Bl}$(a/W), respectively, as given in Fig. 6, it is possible to correct the $J_{u,eff}$ or $J_{0.2Bl,eff}$ values measured on specimens with shallow cracks ("effective fracture toughness") to those which would have been measured if the standard crack-length a_0/W=0.5 would have been used, which are denoted by J_u and $J_{0.2Bl}$, respectively. Consequently, the same correction has to be made to the lower bound values given by (5a) and (5b), if the initial crack size does not meet the requirements of the testing standards.

Thus, combining (5) and (14), the transferable lower bound fracture toughness is obtained by

$$J_{0.2Bl,LB}(a_0/W) = \frac{s_f \cdot W \cdot (1 - a_0/W)}{20 \cdot Y_{0.2Bl}(a_0/W)} \tag{15a}$$

$$J_{u,LB}(a_0/W) = \frac{s_f \cdot W \cdot (1 - a_0/W)}{50 \cdot Y_u(a_0/W)} \tag{15b}$$

The ratio of the transferable lower bound fracture toughness as given in (15) to the values in the case of a standard initial crack length a_0/W=0.5, i.e.

$$X_u(a/W) = \frac{J_{u,LB}(a/W)}{J_{u,LB}(a/W = 0.5)} \quad ; \quad X_{0.2Bl}(a/W) = \frac{J_{0.2Bl,LB}(a/W)}{J_{0.2Bl,LB}(a/W = 0.5)} \tag{16}$$

are shown in Fig. 7. Therefrom one can see that the lower bound fracture toughness is maximum if the initial crack length is about 0.25W.

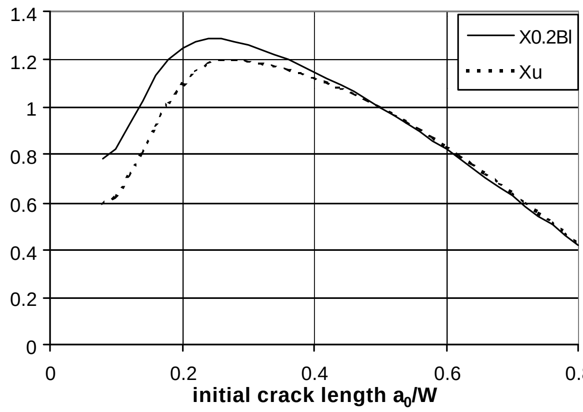


Fig.7: Ratios X_u and $X_{0.2Bl}$ as defined in (16)

Discussion and Conclusions

If shallow cracks are used in fracture toughness testing, then a correction to compensate for the lacking constraints has to be done. A simple correction formula to account for this effect has been shown. If the specimens are sub-sized, then the corrected value still is not a valid (transferable) plane strain fracture toughness, but only a lower bound thereof. Obviously, one usually strives for lower bounds that are as high as possible. According to Fig. 7 these lower bounds are maximum if an initial crack length of about 0.25W is used. Thus, using such shallow cracks instead of the standard 0.5 W is not only advantageous from an experimental point of view, but also leads to less conservative transferable fracture toughness values. This is true for initiation ductile tearing as well as cleavage fracture. Furthermore, beyond the limit of valid J, there seems to be also no significant effect of shallow initial cracks: The dynamic J-R-curve obtained from precracked Charpy specimens with shallow cracks was not clearly different from the ones obtained with deep cracks [14].

Of course, the accuracy of the presented theoretical analysis is limited due to the simplistic models used. The accuracy of the above-mentioned optimum crack-length is hardly better than about ±0.05 W. However, the presented results are rather of qualitative than quantitative relevance. They just indicate that in case of impact testing of sub-sized specimens an initial crack length in the range of about 0.25W to 0.3W performs better than a standard crack. As noted in footnote 3, this conclusion does not entirely rely on the presented simplistic theoretical analysis. The analytically derived curves for constraint correction, which form its theoretical basis, can be simply considered as empirically obtained by fitting the experimental data shown in Fig 6 (b) or some of the numerous further data available in the literature. The over-all conclusions remain the same.

APPENDIX

Analytical Estimation of m for an Edge-Cracked Beam Under Bending

Consider a cracked beam (a/W>0.2) under a bending moment M (Fig. A1) In the case of a relatively small specimen in the state of fully plastic yielding the plastic component of J is much larger than the elastic one, thus

$$J \cong J_{pl} = \frac{\boldsymbol{h} \cdot W_p}{B \cdot b} \quad \text{(A.1)}$$

W_p denotes the plastic work done by M and η the well-known η- factor.

The plastic limit load is given by

$$M_p = \frac{c_p \cdot R_p \cdot b^2}{4} \quad \text{(A.2)}$$

where c_p is a constant in the interval

$$1.26 < c_p < 1.45 \quad \text{(A.3)}$$

the upper boundary holding for a von-Mises-material, the lower for a Tresca-material [18, 16]. For a relative rotation of the crack surfaces by an angular increment d$\boldsymbol{J}$ the energy increment

$$dW_p = M_p \cdot d\boldsymbol{J} \quad \text{(A.4)}$$

is dissipated. The center of rotation of the corresponding relative rotation is assumed to be located in the distance

$$r_{CR} = c_{CR} \cdot b \qquad \text{(A.5)}$$

from the crack-tip. Thus,

$$\boldsymbol{d} = c_{CR} \cdot b \cdot \boldsymbol{J} \qquad \text{(A.6)}$$

Using the relations (see equ. (9))

$$d\boldsymbol{d} = \frac{dJ}{m \cdot R_p} \qquad \text{(A.7)}$$

and (A.1)-(A.6) leads to

$$m\left({a_0}/{W}\right) = \frac{c_p\left({a_0}/{W}\right) \cdot \boldsymbol{h}\left({a_0}/{W}\right)}{4c_{CR}\left({a_0}/{W}\right)} \qquad \text{(A.8)}$$

The behavior of c_p, η and c_{CR} (based on data from [15] and British Standard BS 5762) is shown in Fig. A2. The resulting m(a/W) from (A.8) is also shown in Fig. A.2

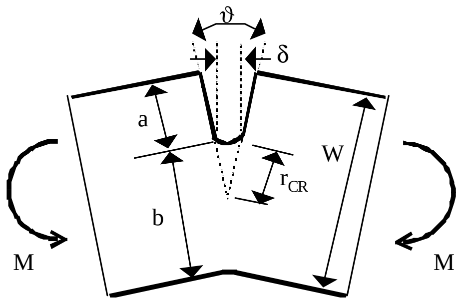


Fig. A.1: Section of an edge cracked beam loaded by a bending moment

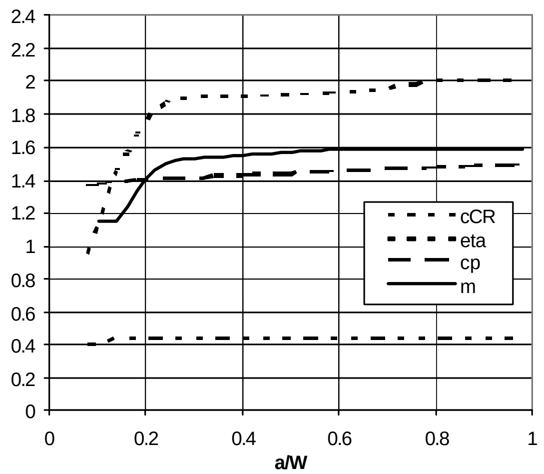


*Fig. A.2: c_p, c_{cr}, **h** and m for an edge cracked beam under bending*

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