Residual Stress Measurement in Cracked Components: Capabilities and Limitations of the Cut Compliance Method

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Abstract: The cut compliance method is a powerful experimental tool to measure residual stress distributions. Based on fundamental theoretical relations of liner-elastic fracture mechanics, it is particularly well suited to be applied to components that contain cracks, since it delivers not only the residual stress-field but the stress intensity factor due to the latter as well. However, in the presence of cracks, there are special problems to be dealt with: Contact between crack faces due to crack closure, large stress gradients near the crack tip, residual stresses as high as the yield stress, problems of cutting and unsteady heat production. Most of these effects disturb the linearity of the problem, so the basic assumptions of the theory are violated to some degree. Some possibilities of how to deal with these problems are proposed and discussed.

Introduction

Structural components that contain residual stresses are likely to contain cracks, too, as well as vice versa, because there is a pronounced interaction between residual stresses and cracks. On one hand the residual stresses promote most mechanisms of crack initiation and sub-critical growth, especially by stress corrosion or fatigue. On the other hand, cracks interfere with residual stress fields. Due to the strain concentration they are a source of additional local residual stresses due to the stress concentration, they cause stress relaxation and redistribution near the crack faces, and additional residual stresses may be produced due to crack closure effects. The latter result from local dilatation that accompany most damage processes, e.g. corrosion products stuck on the crack faces, or the well known crack closure effects in fatigue. Affecting the subsequent growth of the crack, the residual stresses in the cracked section should be known in a theoretical prediction of the crack growth rate. However, measuring them is not an easy task.

A straightforward method to determine residual stress profiles is the cut-compliance method (sometimes also called crack-compliance- or cutting- method), which was proposed and developed by Cheng and Finnie in a series of papers [1, 2] (see [2] for more references) and – among others - further developed by the present author and co-workers [3 – 5]. The idea of the method is to introduce continuously a cut in the body, measuring the resulting stress change and calculating therefrom the released stresses. A good review on measuring residual stresses by continuous cutting is given in a recent paper by Prime [6]. A short description of the cut-compliance-method (abbreviated in the following by CC-method) is given in the next section.

As pointed out in [7, 8] a particularly fruitful area of application of the CC-method is determination of residual stresses in partly cracked cross sections of structural components. Unlike other methods for the same purpose, the CC-method delivers the information about residual stresses in a suitable form for direct use in a fracture mechanics analysis, including the stress intensity factor (SIF) due to the residual stresses. However, some special difficulties are encountered when dealing with cracked bodies, the main ones being contact between crack faces due to closure effects, large stress gradients near the crack tip, high residual stress levels, and possibly problems of non-stationary heat generation due to cutting. Most of these effects are of non-linear nature, so the basic assumption of the underlying theory of linear-elastic fracture mechanics (LEFM) can be violated to some degree. Nevertheless, the CC-method is still applicable in many cases of cracks embedded in residual stress fields, but it is good to be aware of the special problems that are encountered and the corresponding limitation of the method. These are pointed out and discussed in the present paper.

Principle of the CC-Method

A cut has to be introduced along the plane where the residual stresses are to be measured, as shown in Fig. 1 for the case of a rectangular plate with a cut along its central section. If the cut is relatively narrow (d<<a and d<<(W-a)), it can be regarded as a crack, so the basic equations of linear elastic fracture mechanics can be used to establish the required mathematical relation between the residual stress and the strain change at the measurement point. As shown in [3 - 5] the SIF at the tip of the above mentioned cut due to the residual stress (for the sake of simplicity we restrict ourselves to Mode-I-cases and to normal stresses with respect to the cut plane) is given by

$$K_{Irs}(a) = \frac{E'}{Z(a)}\frac{d\varepsilon_M}{da} \tag{1}$$

where ε_M denotes the strain measured at the (arbitrary) location M during the cutting process and recorded as a function of the cut depth a, and E' the generalized Young's modulus (i.e. E'=E for plane stress and $E'=E/(1-\nu^2)$ for plane stain). Z(a), called the "influence function", is a unique function that depends on the component geometry, on the cut plane and on the location of the measurement point M, but not on the residual stress distribution. It characterizes the sensitivity of the measurement point M with respect to the stress release at the cut plane. Determination of Z(a), is the crucial and - considering the theoretical and computational effort - the most demanding step of the CC-method. However, Z(a) needs to be determined only once for a certain geometry and measurement point. Some solutions for Z(a) can be found in [3 – 5].

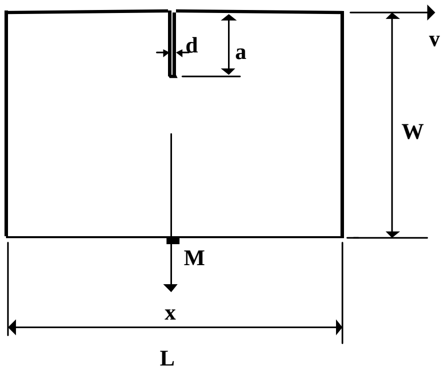

Fig. 1: Rectangular plate, cut along its center plane; strain measurement at point M

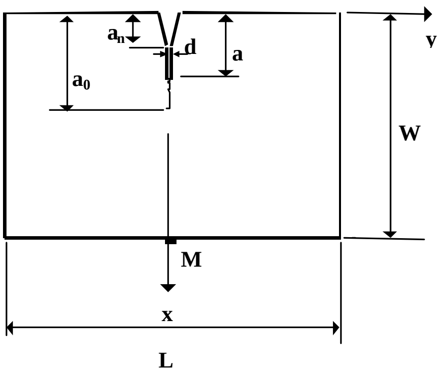

Fig. 2: Application of the CC-method to a rectangular plate containing an edge-crack of length a_0

$K_{Irs}(a)$ results from the normal residual stresses acting prior to cutting, $\sigma_{rs}(x)$ (the x axis being chosen such that it coincides with the crack line, or cut plane, respectively), by the general relation

$$K_{Irs}(a) = \int_0^a h(x,a)\cdot\sigma_{rs}(x)\cdot dx \qquad (2)$$

where h(x,a) denotes the so-called weight function, which is universal for a given crack geometry and available for many systems [9]. $\sigma_{rs}(x)$ can be obtained by inversion of eq. (2). The available mathematical techniques for this purpose are discussed in [6].

Non-linear Effects

Both eq. (1) and (2) require a linear-elastic behavior of the system. In practical application, this condition may be not fulfilled, mainly because of plastic yielding at the cut front and the finite width d of the cut (Fig. 1), which is not a perfect crack in the sense of LEFM. Furthermore, there might be effects of local heating due to cutting. These effects are shortly discussed in the following.

As stated in the previous section and shown below, the effect of the finite cut width is negligible if $d \ll a$ and $d \ll (W-a)$. From this point of view, d should be chosen as small as possible. However, concerning the choice of d, one encounters a. It is well known that the smaller d, the higher the stress concentration at the cut front and, correspondingly, the more likely nonlinear effects due to plasticity or even unstable fracture may occur. From the estimation of the stress concentration of a blunted notch given in [10], it readily follows that plastic yielding at the cut front is absent for

$$K_{Irs} < \frac{\sigma_f}{2}\cdot\sqrt{d} \qquad (3)$$

where $\sigma_f = (R_p + R_m)/2$ (R_m and R_p being the tensile strength and the yield stress, respectively) denotes a representative flow stress of the material. In addition, to avoid unstable crack extension at the cut front, K_{Irs} must be smaller than the critical SIF of the notch (analogous to K_{Ic} in the case of a crack), which is increasing with increasing d.

If the above conditions concerning d and K_{Irs} are not fulfilled, then the CC-method is no longer exact. Nevertheless, in many cases it is still applicable with sufficient accuracy. Furthermore - as in LEFM, where the range of applicability can be extended by introducing a so-called effective crack length [11] - the cut depth a can be replaced by the "effective cut depth" a_{eff}, which accounts for the larger energy-release rate of an extending cut of finite width with a plastic zone in comparison to a perfect crack. We assume a_{eff} to be composed of the physical cut depth a plus a correction, Δa_d, for the finite cut width and another one, Δa_p, for the plastic zone, thus

$$a_{eff} = a + \Delta a_d + \Delta a_p \qquad (4)$$

Using the approximate energy release rate of a blunted crack given in [12] in the derivation of (1), one finds for the case of relatively deep cuts the simple approximation

$$\Delta a_d \cong d \qquad \text{(for } a > 0.2W\text{)} \qquad (5)$$

The second one is assumed to be the classical plastic zone correction of Irwin [11], modified such that it vanishes for K_{Irs} fulfilling the condition (3), thus

$$\Delta a_p \cong \left\langle \frac{1}{2p} \cdot \left[\left(\frac{K_{Irs}}{s_f} \right)^2 - \frac{d}{4} \right] \right\rangle \tag{6}$$

where <•> is the Macauly bracket, which is defined as <•>=0 for •<0 and <•>=• for •>0. Working out mathematically the derivative $d\varepsilon_M/da_{eff}$ in (1) becomes lengthy, so it is omitted here. In practical application, (1) is usually evaluated numerically anyway. It is recommended to do iteratively as follows: As a first step, K_{Irs} is calculated tentatively by (1) without any correction. If condition (3) is heavily violated (a minor violation is acceptable regarding the limited accuracy due to other effects), then (1) shall be re-evaluated by replacing a by a_{eff} as obtained from (4) with K_{Irs} taken from the first step. Therefrom the curve $K_{Irs}(a_{eff})$ results, which can be transformed by (4) back to $K_{Irs}(a)$. If there is a significant change compared with $K_{Irs}(a)$ from the first step, then one more iterative step should be done.

From the correction procedure described above it is evident that the correction effect is relatively small if K_{Irs} does not significantly violate (4) and if $K_{Irs}(a)$ as well as Z(a) have small gradients. This means, that a finite cut width d and a small plastic zone do not much affect the calculated $K_{Irs}(a)$ and $\sigma_{rs}(x)$, since the derivative in (1) is not sensitive with respect to stationary irregularities, at least in regions where the gradients of $K_{Irs}(a)$ and Z(a) are relatively small. In such cases, d and the plastic zone just do some averaging of the stresses within an area of about $(a_{eff}-a)^2$. At present time, no systematic investigation of the benefit of this correction has been performed yet.

One also has to be aware that the measured $\varepsilon_M(a)$ can be affected by the local heat generation due to the cutting process. It has been shown experimentally in [13] that cutting procedures that are not stationary can significantly contribute to the measured strain gradient. If the temperature distribution is stationary, then there will be only a minor effect, for the same reason as for a stationary plastic zone as discussed above.

Application of the CC-Method to Cracked Components

Principally, the CC-method is well suited to be applied to components that contain cracks embedded in a residual stress field. An example of a plate containing an edge crack of length a_0 is shown in Fig. 2. However, the measurement is somewhat more difficult, since a cracked section is more likely to exhibit nonlinear features as discussed in the previous section. The reasons for these additional difficulties are the high stress gradients due to the pronounced stress concentration, the high SIF due to residual stresses action concentrated right at the crack-tip, and the high residual stress level in the reversed plastic zone, which in general reaches or even exceeds the compressive yield stress. A further type of nonlinearity enters the problem in cases of crack closure, which give rise to an elastic contact problem, which is well known to be a nonlinear one. For $a>a_f$, the residual stresses in the ligament are released and obtained by (3). Finally the above mentioned problem of non-stationary heating due to cutting seems to be more pronounced in cracked bodies, because the resistance to cutting depends to some degree also on the residual stress and the degree of closure [13]. These special nonlinear features are discussed in the following.

Crack Closure Effects. If a cut is introduced along crack surfaces that are under crack closure, the closure stresses are released like the residual stresses . Unlike the redistribution of residual stresses due to cutting, the one of contact stresses is governed by non-linear equations, since the elastic stiffness of the body depends on the contact area. Thus, eq. (1) and (2), which are based on LEFM, are no longer valid if the cut is shorter than the initial crack, i.e. $a<a_0$,. This means, that for $a<a_0$ the SIF obtained by (1) and especially the residual stresses obtained from (2) do not give the real initial

distribution of closure stresses. In this respect, the statements concerning the measurability of closure stress distributions made in [7] and [8] appear to be too optimistic. In [7] an example is shown where the peak stress was found to be not at the crack-tip, but just at the crack initiation at the initial notch root. Probably the main reason for this unexpected distribution of the closure stresses is the above mentioned nonlinearity of the closure stress effect.

However, the distribution of closure stresses is not relevant in a fracture mechanics analysis. The closure parameter of primary interest rather is the total SIF due to crack closure, K_{op}. This value is correctly obtained from (1) evaluated at $a=a_0$, i.e. $K_{op} = K_{Irs}(a=a_0)$. The residual stresses in the ligament for $a>a_0$ are also correct.

Stress gradients. As explained in the previous section, systems with large gradients of Z(a) and $K_{Irs}(a)$ are more likely to be influenced by the finite d and the plastic zone at the cut front. It is well known from fracture mechanics that if a crack is loaded to a SIF K_I, then a plastic zone of the length of about

$$r_p = \frac{\pi \cdot}{8}\left(\frac{K_I}{\sigma_f}\right)^2 \qquad (7)$$

is formed. Upon unloading, reversed plastic yielding occurs in a zone that has a length of about

$$r_{pr} = \frac{\pi \cdot}{32}\left(\frac{K_I}{\sigma_f}\right)^2 \qquad (8)$$

i.e. $r_{pr} \cong r_p/4$. Thus, for theoretical reasons, the residual stresses are expected to change from $-\sigma_f$ (yield stress in compression) at $x=a_f+r_{pr}$ to about $\sigma_f/3$ at $x= a_f+r_p$ (see Fig. 3). Depending on the magnitude of K_I and the flow stress, this dramatic change of the residual stresses occurs in a distance of a few millimeters.

Stress gradients in z-direction are due to the triaxiality conditions that are known to vary along the z-axis, and the shape of the crack front, which is in general curved rather than straight. Concerning the stress gradients in z-direction, the CC-method averages the stresses and the SIF.

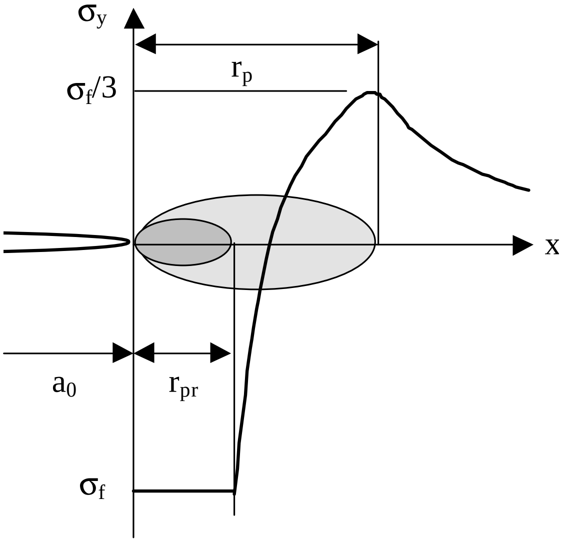

Fig. 3: Distribution of residual stresses in the vicinity of a pre- and unloaded crack tip (schematic)

Reversed Plasticity. If the cracked body has been loaded in tension followed by unloading, there is a zone in the vicinity of the crack-tip that contains residual stresses up to the compressive yield stress, as shown in Fig. 3. The SIF due to these stresses is given by

$$K_{Irs}(\Delta a) = -2\sqrt{\frac{2}{\pi}} \cdot \sigma_f \cdot \sqrt{a-a_0} \qquad (9)$$

As shown in [7] the SIF delivered by (1) in the vicinity of the crack tip can rise much steeper than theoretically predicted by (9). Correspondingly, the stresses calculated by (2) can exceed the yield stress significantly, which is physically not possible. As done in [7], this behavior can be explained by the high stress level in this region: Since the material surrounding the cut front already is in a state of "almost-yielding", there is - so to say - no reserve left for an additional plastic zone to be formed at the cut front. Thus, local plastic yielding and the corresponding stress rearrangement takes place to a much higher extent than expected. Therefore the strain change measured at M is due to the stresses released directly by cutting as well as to the ones released by the accompanying local plastic yielding. This is the reason why eq. (2) leads to an overestimation of the SIF and of the residual stresses in the very first phase of cutting in the compressive plastic zone. To correct approximately for this effect, the calculated K_{Irs}(a)-curve can be modified such that its initial (negative) rising part follows the function given by (9).

Temperature effects. The materials resistance against cutting seems to depend on the closure stress, thus varying along the cut. Thus, the temperature field generated by cutting (which may be EDM or mechanical sawing) is likely to be not stationary, thus influencing the strain gradient. Therefore it is recommended that cutting is performed stepwise by interrupting for the strain measurments, with strain readings taken only after a due waiting time to allow the temperature field to homogenize.

Conclusions

- A finite cut width and a limited amount of plastic yielding is acceptable in the CC-method.
- The range of applicability may be extended by introducing an effective cut length a_{eff} and use it in (1) instead of a.
- Cracked bodies are more likely to exhibit nonlinear effects. Eventually, some special corrections are required.
- In cracked bodies, non-stationary temperature fields are likely to occur. Incremental cutting and strain measurement is recommended rather than continuous cutting.
- Crack closure stress distributions are hardly measurable by the CC-method.
- The crack closure SIF K_{op} can be obtained by the CC-method

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