Near-Surface Stress Measurement in 2D and 3D by the Cut Compliance Technique

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Abstract. The cut-compliance (CC-) method can be applied in a semi-destructive way by introducing only a shallow slot instead of a cut through the thickness. This requires the influence function to account for the U-shape of the shallow slot and the 3D-effects that affect the stress relaxation. In the present paper a simplified way to evaluate surface stresses from a slot is presented and discussed. By examples it is shown that a two-parameter approximation is often sufficient to characterize the near surface residual stresses in a narrow range of depth. Beyond this range, the procedure described herein can be matched with the common procedure of the CC-method. The analysis is explained first for the simpler case of a 2D-structure. Then the generalization to a 3D-case is shown and discussed. For both cases illustrative examples of measurements are given.

Introduction

The cut-compliance or crack-compliance (CC-) method [1 - 4] is more and more recognized as a powerful tool to measure residual stresses. Its main advantages are the capability to determine the residual stress-distribution on the entire cross-section of a structural part by only one or a few strain-gages, and to deliver the residual stresses in a form well-suited for their use in engineering stress- and safety analyses of structures, including the stress-intensity factors acting on cracks [4, 5]. The main disadvantage is its nature as a destructive method, since it requires a cut along the considered cross-section. It also has to be mentioned that the method is practically restricted to 2D structures, because the determination of the general relaxation behaviour of an elastic body in 3D due to a cut is rather complex from a theoretical point of view. This drawback can be circumvented by measuring in two steps: In a first step, a plane plate (e.g. a beam) is cut from the 3D-body and the global stress resultants (normal force and bending moment) are evaluated. As second step, the CC-method is applied to the plane part to deliver the detailed through-the thickness stresses. The total stresses are obtained as a superposition of the corresponding global and local components.

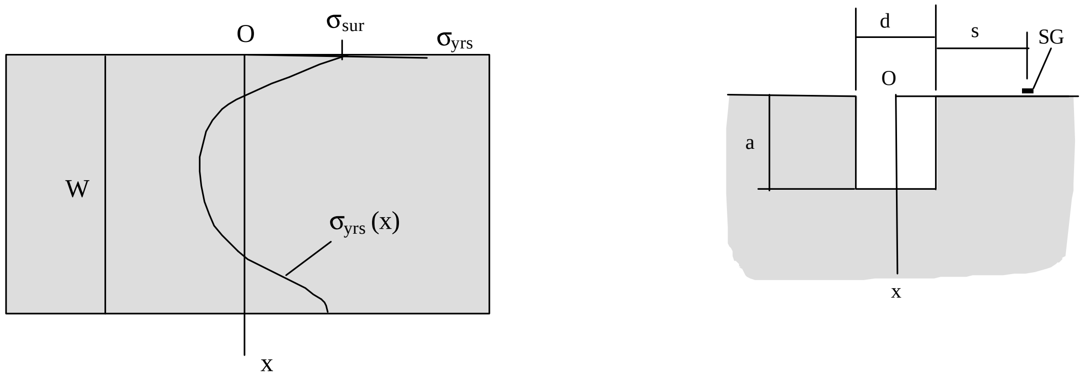
In engineering practice there often is a need to measure or estimate residual stresses in a non-destructive or semi-destructive way directly on the structure, in-situ. Principally, the cut-compliance technique can be applied in a semi-destructive way by introducing only a shallow slot instead of a through-cut [. In this case, of course, only the stresses up to a depth of the slot can be measured or estimated. However, the near-surface stress is often of main interest in an engineering safety analysis, because they interfere with the surface cracks that are usually crucial for the service life of a component.

There are two main difficulties that have to be dealt with when using the CC-method by just a shallow surface slot: First, the latter can mathematically no longer be regarded as a crack [7] and second, the relaxation of the stress due to the cut is influenced by 3-D-effects. These effects have to be accounted for in the compliance function.

In the present paper a relatively simple way to evaluate surface stresses from a slot is presented. The analysis is explained first for the simpler case of a 2D-structure. Then the generalization to a 3D-case is shown and discussed. For both cases illustrative examples of practical measurements are given.

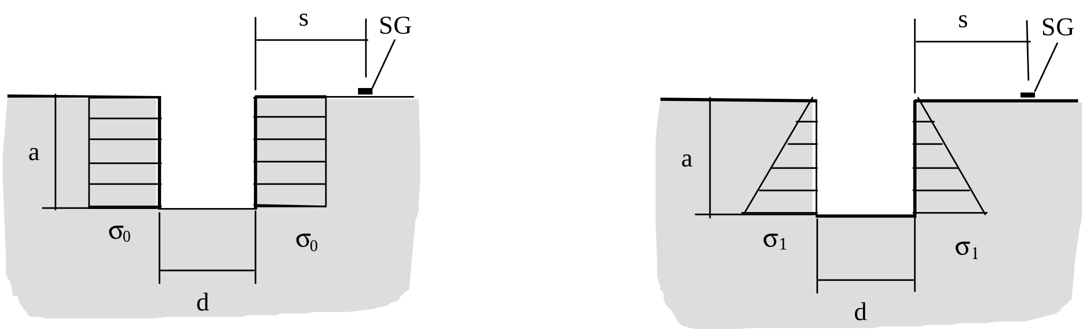
Surface cut in 2D

Consider an arbitrarily shaped 2D-body that contains residual stresses $\sigma_{yrs}(x)$ (Fig. 1a). According to the principal of the CC-method, a cut of width d is progressively introduced at location O, where the residual stress shall be measured (Fig. 1b). The strain change ε_M due to the cut is measured as a function of the slot depth a in a certain distance s from the cut edge by a strain-gage (SG). From this information it is possible to calculate the residual stress $\sigma_{yrs}(x)$ in the range $0<x<a$, provided the theoretical relation between the released stress and the resulting strain, the so-called compliance function, is known in a suitable form. For a U-shaped surface slot this relation is established in the following.



(a) (b)

Fig. 1: Arbitrary 2D-body containing residual stresses (A) and details of the slot introduced at location O and the strain measurement at SG (B).



Load-case 1 Load-case 2

Fig. 2: Basic load -cases

To characterize the stresses near the surface it is often sufficient to approximate them by the surface stress σ_{sur} and the corresponding gradient g_{sur} defined as

$$\sigma_{sur} = \sigma_{yrs}(x=0) \quad (1a)$$

$$g_{sur} = \frac{d\sigma_{yrs}}{dx}(x=0) \quad (1b)$$

These two quantities have to be identified from the strain measured at SG by the strain-gage. In order to establish the theoretical relation between σ_{sur} g_{sur} on one hand and ε_M on the other, two basic load-cases of the plate with the slot are considered (Fig. 2): A constant pressure σ_0 acting on the lateral slot surfaces (load case 1) and a pressure σ_1 linearly increasing with x (load case 2). The relaxation of any stress distribution due to the slot can be expressed as a superposition of these two basic load cases.

To express the stress σ_M at a distance s from the cut in terms of σ_0 and σ_1 a number of numerical stress computations by the boundary element method (BEM) for various parameter sets a, d, s and W were performed. It was found, that σ_M is rather insensitive with respect to d (if a>d/3) as well as to W (if a<W/5), so the influence of these parameters are neglected in the first approach presented here. The resulting stress σ_M(a, s) for the two considered load-cases are shown graphically in Fig. 3. As a polynomial fit they read

$$y_0 = 0.0119x^6 - 0.2778x^5 + 1.4884x^4 - 3.2525x^3 + 2.7959x^2 + 0.1555x \qquad (2a)$$
$$y_1 = 0.034x^6 - 0.3198x^5 + 1.1526x^4 - 1.9272x^3 + 1.2745x^2 + 0.0789x \qquad (2b)$$

where

$$y_0 = \frac{s_M}{s_0} \qquad y_1 = \frac{s_M}{s_1} \qquad x = \frac{a}{s} \qquad (2c)$$

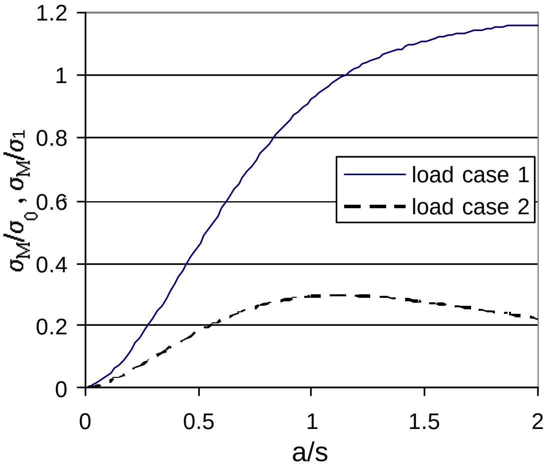


Fig. 3: Stress at a distance s from the cut edge for the two load-cases shown in Fig. 2.

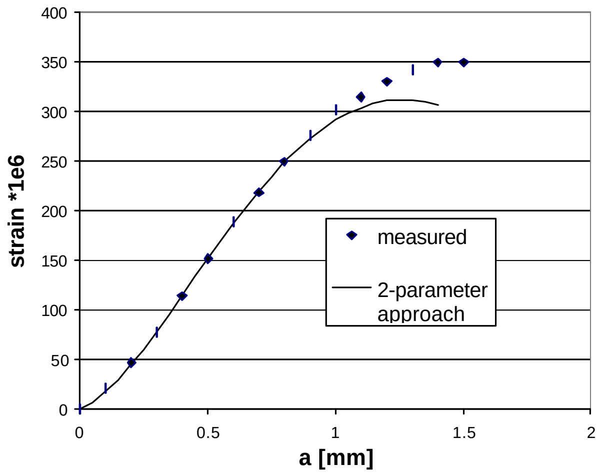


Fig. 4: Comparison of measured strain with strain calculated by (2) with (3) (example)

By means of Eqs. 2 it is possible to determine σ_0 and σ_1 from two measured stresses σ_M at two different cut depths a. An example is shown in Fig. 4. A cut of width d=0.4 mm was introduced in a beam made of a quenched and tempered steel of width W= 20mm. The strain measured at a distance s=2.65 mm in the range 0<a<1.5 mm is shown in Fig. 4. From the stress values at a=0.3 mm and a=0.8 mm the surface stress and its gradient were determined by Eq. 2 to be

$$\sigma_{sur}=-407 \text{ N/mm}^2 \text{ and } g_{sur} = 494 \text{ N/mm}^3 \qquad (3)$$

To check the range of validity of this 2-parameter approximation, the strain calculated by Eqs. 2 is compared with the measured one in Fig. 4. From the coincidence one can tell that the 2-parameter approach matches the actual residual stress distribution near the surface reasonably well up to a depth of about x=1 mm. Beyond this range, the step-by-step inversion technique [5, 8, 9, 10] has to be applied. If there are, like in the shown example, more than the minimum two measured strain values available, the accuracy of the calculated σ_{sur} and g_{sur} can be somewhat improved by using a least-square fit procedure.

Generalization to 3D

Principally the same approach as explained above for 2D-problems can be used to determine the residual stresses near the surface of a 3D-body by introducing a local slot. In this case the cut is usually of a finite length D measured on the surface, so the elastic problem becomes three dimensional features (Fig. 5). The 2D-approach can be applied only if a<<D and s<<D. To minimize the damage one usually strives for a relatively small D, so the three dimensional effects have to be taken into account, including the shape of the cut.

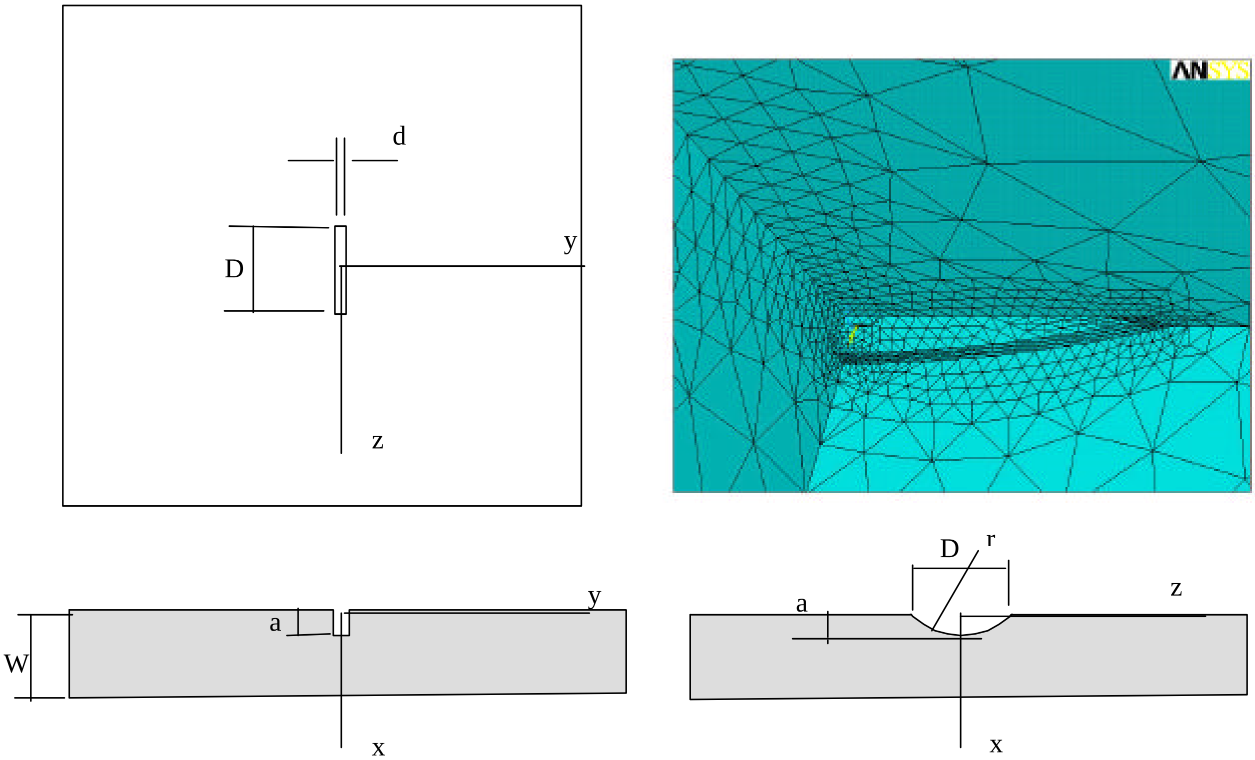


Fig. 5: Circular surface cut introduced in a plate and example of a FEM-model to calculate the stresses for the analogous basic load-cases 1 and 2 shown in Fig. 2.

To introduce a cut in the surface of a component suitable means are a circular saw or a grinding disk. Both produce a slot with a cylindrical front as shown in Fig. 5. Analogous to the procedure described in the previous section for 2D, cuts of various depths a are analyzed for the two load-cases (1) constant pressure σ_0 and (2) linearly increasing with x to σ_1 (see Fig. 2). In the FEM-analysis it is preferable not to consider the stress σ_M at the strain gage location, but the strain ε_M, thus $\varepsilon_M = \varepsilon_y(x=z=0, y=s+d/2)$, because the stress-strain relation is affected by the stress triaxiality. For load-case 1 (uniform pressure σ_0 acting on the lateral cut surfaces) the FEM-results are shown Fig. 6. Assuming s=3 mm as used in the experiments performed, the corresponding intervals of these curves are fitted to

$$y = -0.1359x6 + 1.0441x5 - 3.2742x4 + 5.1593x3 - 3.682x2 + 0.0449x \quad (4)$$

where $y = \frac{\varepsilon_M \cdot E}{\sigma_0}$ and $x = \frac{a}{s}$ (4a)

As shown in Fig. 6 this polynomial is in relatively close agreement with the plane-strain curve corresponding to (2a) in the relevant range 0.1<a/s<2. The deviation from the 2D (plane strain) curve is shown in Fig. 7 for some cut depths a. These correction curves are expected to be dependent on the ratio s/r, where r is the radius of the cutting tool (Fig. 5). In the present case it was chosen to be r=18.5 mm according to the grinding disk used to introduce the cut.

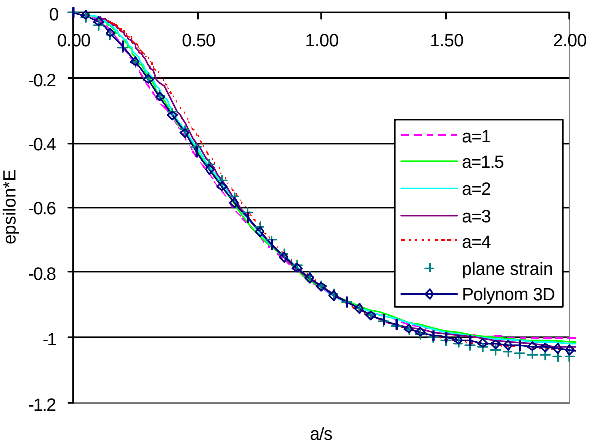


Fig. 6: Strain at distance s for load-case 1 (σ_0=1) for different depths a

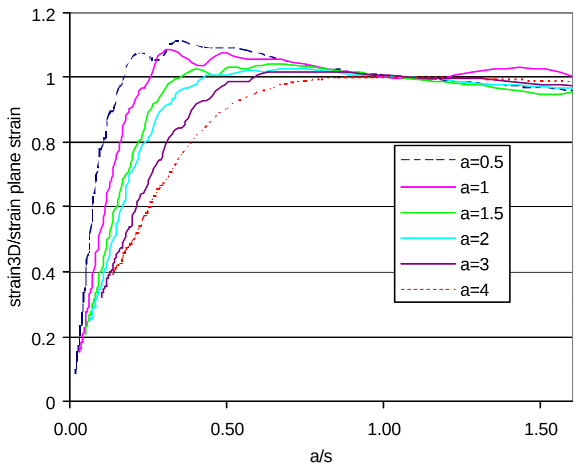


Fig. 7: Ratio of strain ε_M in 3D to ε_M in plane strain for some cut depths a

Concerning load-case 2 it is interesting to note that the ratio of load-case 2 to load-case 1 in terms of stress σ_M or strain ε_M is approximately a linear function of a/s in the range 0.1<a/s<2, as shown in Fig. 8. Note that as a/s is decreasing a is decreasing, so the upper bound is relevant. The following linear fit holds as an approximation of the upper bound for 0<a/s<2:

$$\frac{\sigma_M(\sigma_0=1)}{\sigma_M(\sigma_1=1)} \cong \frac{\varepsilon_M(\sigma_0=1)}{\varepsilon_M(\sigma_1=1)} \cong 0.4704-0.1493\cdot\frac{a}{s} \qquad (5)$$

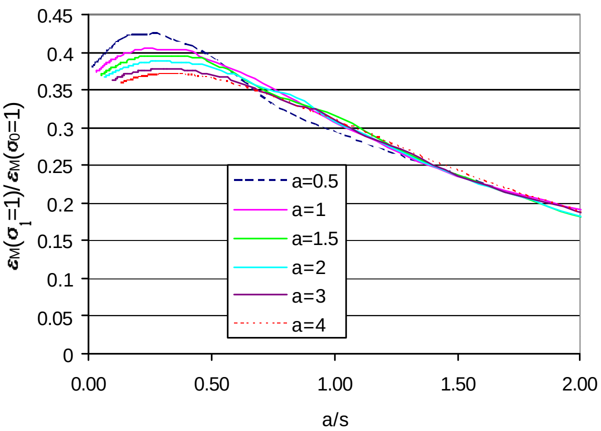


Fig. 8: Ratio of ε_M of load-case 2 (σ_1=1) to ε_M of load-case 1 (σ_0=1).

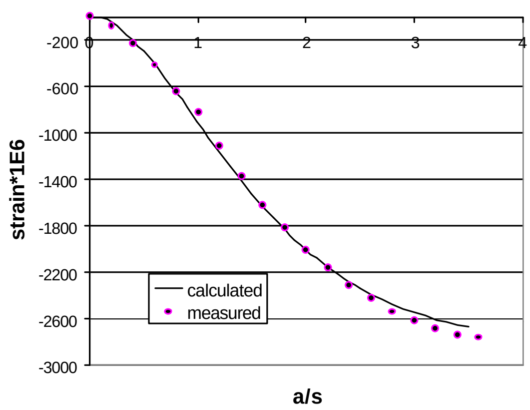


Fig. 9: Comparison of measured strain and strain calculated by (4) and (5) with the values (6)

The surface stress σ_{sur} and its gradient g_{sur} can be determined from two strain measurements at two different cut depths analogously to the procedure explained above for 2D-situations by Eqs. (4) and (5). As a practical example, the procedure was applied to a high-strength steel structure near a weld.

The strain measured at s=2.65 mm as a function of a is shown in Fig. 9. Therefrom the near-surface residual stress was determined to be

$$\sigma_{sur}=710 \text{ N/mm}^2 \text{ and } g_{sur} = -160 \text{ N/mm}^3 \quad (6)$$

The relatively good agreement between the calculated and the measured strains indicate that the used two-parameter approximation describes the actual residual stress-distribution fairly well up to a depth of about 2.5 mm.

Discussion and Conclusions

It is shown how the CC-method can be applied in a simplified way as a semi-destructive method to determine the near-surface residual stresses. Although the damage done to the structure is somewhat more severe compared with the one produced by the hole-drilling method, it often can be repaired still satisfyingly. On the other hand the sensitivity of the CC-method is better, and more information is obtained than by using the hole-drilling method. Furthermore, it can be applied even to hard materials, where drilling is no longer possible.

The approximation of the near-surface stress by two parameters restricts its use to a near-surface zone. The range of validity can be checked by comparing the calculated and the measured strain values as shown by examples in this paper. Beyond the range of coincidence, the stress distribution has to be evaluated according to the principles of the CC-method as described in [5, 8-10]. The two methods can be easily matched, as the values of σ_{sur} and g_{sur} serve well to determine the starting values that are required by the step-by-step inversion technique described in [5, 8-10].

The good agreement between the strains in 3D and in plane strain indicate that the influence function known for the front strain-gage of a surface crack in 2D holds as an approximation also for the 3D-case. Thus, beyond the surface layer which is covered by the two-parameter approach discussed above, strain measurements of 3D-cuts can be evaluated to determine the stress distribution by the same inversion technique as described in [5, 8-10], using the same influence function as an approximation. This holds if s<<D and if the residual stresses are rather homogeneous in lateral direction.

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