An Engineering Framework to Account for Crack-Tip Constraints in Fracture Mechanics

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1. Introduction

Stress concentration and inhomogeneous plastic deformation in the vicinity of a crack tip cause a triaxial stress state to occur in this region. The magnitude of the crack-tip stress triaxiality is dependent on several parameters like the plate thickness, the crack size and the geometry and loading conditions of the component. Its effect on ductility and fracture behavior is responsible for the well known "embrittlement" of the material in presence of a crack. These so-called crack-tip constraints should be taken into account in failure assessment analysis of structural components. Conceptually it is suitable to distinguish between out-of-plane constraints (OPC) and in-plane constraints (IPC). The importance of the former is known since the early days of fracture mechanics, when it was found that fracture toughness is significantly dependent on the thickness of the test specimen [1, 2]. It took about twenty years longer until the effect of the IPC on fracture toughness was recognized [3-8].

To characterize the IPC several parameters are suggested in the literature. The best known are the so-called T-stress, the Q-factor and the m-factor. The T-stress quantifies the second term of Williams stress field expansion [9, 10], which - as a homogeneous uniaxial stressfield in crack-direction - obviously contributes to the triaxiality of the near-tip region, at least for small scale yielding (SSY). Moreover - surprising on a first glance - there is experimental and numerical evidence that T correlates with the constraint effects even in the case of large scale yielding (LSY) and full scale yielding (FSY) [7, 8, 9]. However, in LSY or FSY, T is expected to serve only as an "indexing" parameter, not a "correcting" one (in the sense of [7]).

The Q-factor [5] quantifies the difference between the actual local stress at a certain reference location near the crack tip and the theoretical HRR-stressfield under SSY-conditions. A detailed local stress analysis by the Finite-Element method (FEM) is required for its determination. So - since the main benefit of engineering fracture mechanics just is its possibility of predicting the local fracture behavior at a crack tip without a detailed local stress analysis - the Q-approach is often not adequate in practical engineering applications.

The factor m is well known as a constraint-dependent factor in the linear relation between J and CTOD (see eq. (5) below). It obviously reflects the local ductility and, therewith, the constraints, so it is suggested in [11] to be used directly as a constraint parameter. However, it also seems to be rather an indexing than a correcting parameter.

Fig. 1 shows schematically the distribution of the stress σ_y in y-direction in the vicinity of a crack-tip in an elastic-plastic material. The constraints affect first of all the local maximum stress, σ_{ymax}, which plays a key role in the fracture processes (see section 4). Thus, the most direct parameter to quantify the crack-tip constraints is the factor γ defined as

$$\gamma=\sigma_{ymax}/R_p \qquad (1)$$

The drawback of γ as a constraint-parameter also is the difficulty of its determination. Like Q, it also requires a local, nonlinear 3D-FEM-calculation, making it unsuitable for practical use in a standard fracture mechanics application.

The present investigation is an attempt to simplify characterization of constraints for engineering applications. For this purpose an approximation of γ, denoted by γ_{mT}, is suggested. Although considerably easier to determine, it still enables one to estimate constraint effects on fracture behavior of a component with a reasonable accuracy.

2. Estimation of Maximum Stress

To obtain an approximation of γ, the maximum local stress σ_{ymax} as defined in Fig. 1 is estimated. For SSY and elastic-perfectly plastic material under plane strain conditions, the local stress was obtained in [12] by slip line theory to be

$$\sigma_{y\,max} = 0.5 \cdot (2+\pi) \cdot R_p \qquad (2)$$

where R_p denotes the yield stress. Eq. (2) holds for Tresca's yield criterion; for the von-Mises yield criterion the factor 0.5 has to be replaced by 0.577. By the same model, the relation between J-integral and the crack-tip opening displacement δ was obtained in [12] to be

$$\delta = \frac{4}{(2+\pi)} \cdot \frac{J}{R_p} \qquad (3)$$

From eq. (2) and (3) it follows that

$$\sigma_{ymax} \approx 2 \cdot m \cdot R_p \qquad (4)$$

where m is the factor appearing in the well known general relation

$$J = m \cdot R_p \cdot \delta \qquad (5)$$

The stress corresponding to a slip line field can be altered by a superimposed hydrostatic stressfield. In SSY, such an additional stressfield is risen by the T-stress. In a simplistic overall view, the T-stress causes in the elastic field surrounding the plastic zone the following relation:

$$\sigma_x = \sigma_y + T \qquad (6a)$$

Plane strain condition and Tresca's yield criterion require

$$\varepsilon_z = \frac{1}{E} \cdot (\sigma_z - \nu \cdot (\sigma_x + \sigma_y) = 0 \qquad (6b)$$

$$\sigma_y - \sigma_z < R_p \qquad (6c)$$

(6a) - (6c) lead to

$$\sigma_{y\,max} = \frac{R_p + \nu \cdot T}{1 - 2\nu} \qquad \text{(for T<0)} \qquad (7)$$

Roughly, the first term in (7) corresponds to (2), thus one can write

$$\sigma_{ymax} \approx 2 \cdot m \cdot R_p + \frac{\nu}{1-2\nu} \cdot T \quad \textit{for plane strain, } T<0 \qquad (8a)$$

$$\sigma_{ymax} \approx 2 \cdot m \cdot R_p \quad \textit{for plane strain, } T>0 \qquad (8b)$$

For plane stress, T has no influence on σ_{ymax} and the lateral necking results in a strip-yield-type behavior with

$$\sigma_{ymax} \approx R_m \quad \textit{for plane stress} \qquad (8c)$$

and R_m being the tensile strength.
The above considerations hold for SSY only. Nevertheless, there are some physical reasons to assume, that (8a) - (8c) can be extended to estimate the local stress well beyond SSY. The factor m, which is the dominating term in (8a), is defined in LSY and FSY as well, and it is obviously related to constraints in these cases, too. As mentioned above, the elastic T-stress also has some significance in LSY and FSY.

3. Definition of the Constraint Parameter γ_{mT}

Partly based on (8a) - (8c) and the above discussed extension to LSY, partly just on intuition, a constraint parameter γ_{mT} is tentatively defined as a linear combination of m and T in the following form:

$$\gamma_{mT} =: c_m \cdot m + c_\beta \cdot \beta \qquad (9)$$

where

$$\beta = T_{max}/R_p \qquad (10)$$

with T_{max} being the maximum T-stress at the considered crack, i.e. the T-stress at $K_I = K_{Ic}$ for SSY or for the plastic limit load in LSY and FSY, respectively. According to the considerations discussed above the factors in (7) are

- for plane strain, T≤0 : $c_m = 2 \quad c_b = 0.75$ (11a)
- for plane stress: $c_m = \sigma_f / R_p \quad c_b = 0$ (11b)

γ_{mT} is expected to be an approximation of γ. It is considerably simpler to be determined, since both its ingredients, T and m, are well known parameters, that can be found in the literature for several systems, at least as approximations, or which can be calculated relatively easily. Actually, determination of m according to (5) requires a FEM-calculation, but a relatively coarse net is sufficient to calculate determine J and δ. Unlike Q, it also can be determined experimentally or - as in the example in section 5 - even analytically.

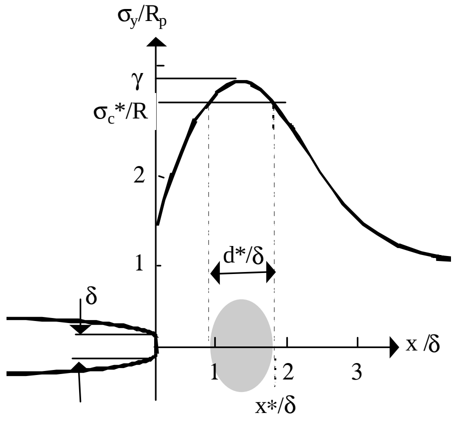

Fig. 1: Non-dimensional representation of the stress distribution in the vicinity of a crack-tip

4. Estimation of Constraint Effects on Fracture Toughness

In the following relations g can be generally replaced by its approximation, g_{mT}.

4.1. Cleavage Fracture

The J- or CTOD-values at onset of cleavage are denoted by J_c and δ_c, respectively. Both are known to be constraint-dependent. In the following, this dependence is estimated. Taking pattern from [13], the following two criteria are assumed to govern initiation of unstable cleavage:

i) The maximum stress in the vicinity of the crack tip must exceed the cleavage stress σ_c*, i.e.

$$\sigma_{ymax}= \gamma \cdot R_p > \sigma_c^* \qquad (12)$$

ii) The elastic energy $W_{el} = \int U_{el}\, dV$ stored in a critical Volume V* of the width d* (Fig. 1) must be sufficient to produce a cleavage fracture in the area 0<x<x*.

Using the proportionalities

$$U_{el} \propto (\gamma \cdot R_p)^2\ ;\ V^* \propto \delta^2,\ \ d^* \propto \delta\ ;\ \ x^* \propto \delta$$

and the general relation (5) in criterion ii), one finds the proportionality

$$J_c \cdot \frac{g^2}{m} = const \qquad \text{for } \gamma > \sigma_c^*/R_p \qquad (13)$$

By using (5), the analogous proportionality can be written in terms of CTOD, i.e.

$$d_c \cdot g^2 = const \qquad \text{for } \gamma > \sigma_c^*/R_p \qquad (14)$$

For $\gamma < \sigma_c^*/R_p$ no cleavage occurs.

4.2. Ductile Tearing

The J or CTOD- values at or near initiation of ductile tearing are denoted in the following by J_{it} and δ_{it}, respectively. They represent near initiation parameters such as $J_{0.2}$, $J_{0.2/Bl}$, J_{Ic} , etc., or the corresponding CTOD-values, respectively. J_{it} and δ_{it} are size independent only if the corresponding size requirements are met [14] and if the crack-tip constraint of the test specimen is as high as in the standard test specimens, i.e. deep cracked bend or CT specimens. Otherwise they are constraint-dependent. This effect is estimated in the following.

We assume that δ at crack initiation is proportional to the plastic failure strain ε_{pf} in the fracture process zone,

$$\delta_{it} \propto \varepsilon_{pf} \qquad (15)$$

ε_{pf} is known to be constraint-dependent [15]. Based on the failure hypothesis of Gillemot [16], which states that ductile failure occurs when the plastic energy density U_p reaches a certain critical value U_{pf}, we simply assume that the product of the true failure strain and $\sigma_{ymax} = \gamma \cdot R_p$ is constant at crack initiation, i.e.

$$g \cdot R_p \cdot ln(1+e_{pf}) = U_{pf} \qquad (16)$$

The logarithmic strain is used because ε_{pf} is in general not small enough to be linearized. According to [17] U_{pf} can be roughly obtained from a uniaxial tensile test as the area under the true stress-true strain diagram in the necking area, which is approximately

$$U_{pf} @ \frac{s_f \cdot Z}{1-Z} \qquad (17)$$

where $\sigma_f=(R_p+R_m)/2$ denotes the flow stress and Z the standard reduction of area. With (16) and (17), (15) leads to

$$\frac{d_{it}}{exp\left[\frac{s_f \cdot Z}{R_p \cdot g \cdot (1-Z)}\right]-1} = const \qquad (18)$$

Using (5) in (18) delivers the proportionality in terms of J:

$$\frac{J_{it}}{m \cdot \left\{ exp\left[\frac{\sigma_f \cdot Z}{R_p \cdot \gamma \cdot (1-Z)} \right] - 1 \right\}} = const \qquad (19)$$

5. Examples

5.1. Edge Cracked Beam Under Bending

For edge-cracked beams under bending (Fig. 2) the constraints are known to depend on the crack-length [7, 9, 18, 19]. For this reason this system is used to check the validity of γ_{mT} as a constraint parameter. Under FSY the following relation was analytically obtained in [20]:

$$m = \frac{c_p \cdot \eta}{4 c_{CR}} \qquad (20)$$

Herein, $c_{CR} = r_{CR}/b$ is the non-dimensional distance between the crack-tip and the center of rotation, and η the well known eta-factor that relates J to plastic energy. c_p denotes the plastic constraint factor that appears in the plastic limit load as

$$M_p = \frac{c_p \cdot R_p \cdot b^2}{4} \qquad (21)$$

The values of these parameters are taken from the literature (Fig. 3). The parameters m according to (20) and β as defined in (10), calculated from the T-stress given in [8] at the plastic limit load (21), are shown in Fig. 4, as well as the resulting γ_{mT}.

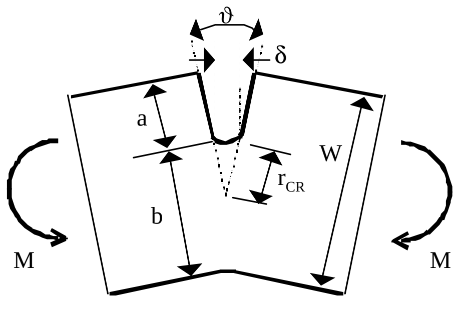

Fig. 2: Edge-cracked beam under a bending moment M

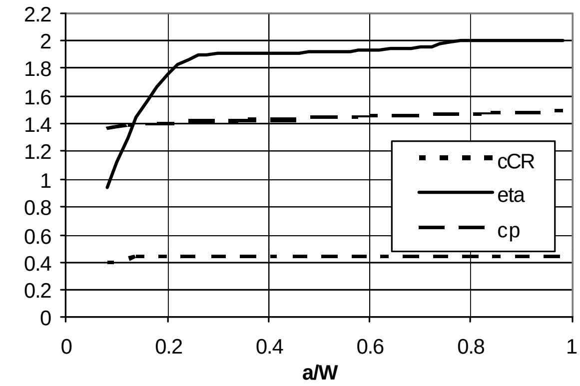

Fig. 3: Factors appearing in eq. (20), as functions of a/W (c_p from [21], η from [19]; c_{CR} represents the trend extracted from [18 and 22].)

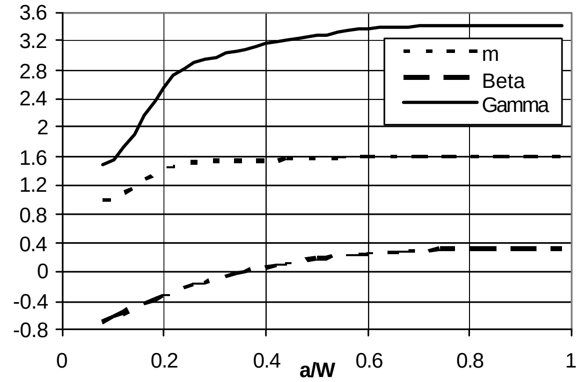

Fig. 4: m, β and γ_{mT} as a function of crack length

γ_{mT} inserted in (13), (14), (18) and (19), respectively, gives the prediction of the constraint effect on the apparent fracture resistance in terms of J or δ. Fig. 5 shows the effect of crack length on the apparent fracture toughness. These predictions are compared in Fig. 6 and 7 with experimental fracture toughness data reported in literature. Comparison of cleavage data reported in [7] with the J_c- curve in Fig. 5 exhibits a similar agreement. Regarding the typical scatter in the experimental data, the agreement between predicted and experimental data is satisfactory.

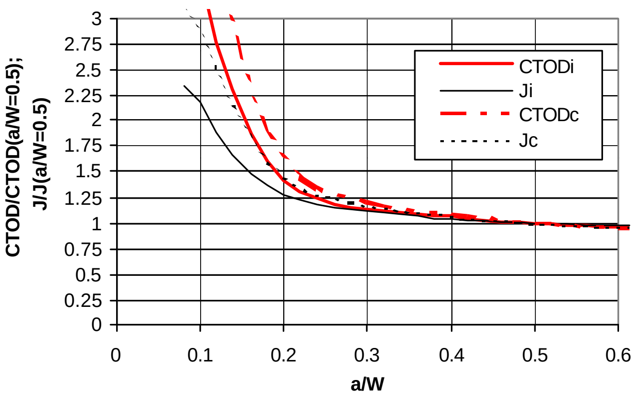

Fig. 5: Ratios of apparent fracture toughness to the corresponding values at a/W=0.5 as a functions of non-dimensional crack length.

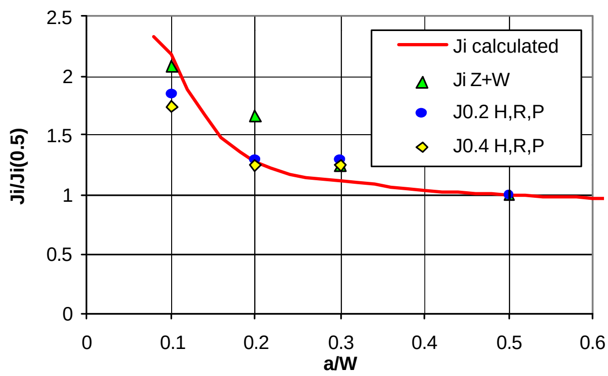

Fig. 6: Comparison of J_{it} predicted by (19) with experimental J_{it} (Z+W from [18], H,R,P from [9]).

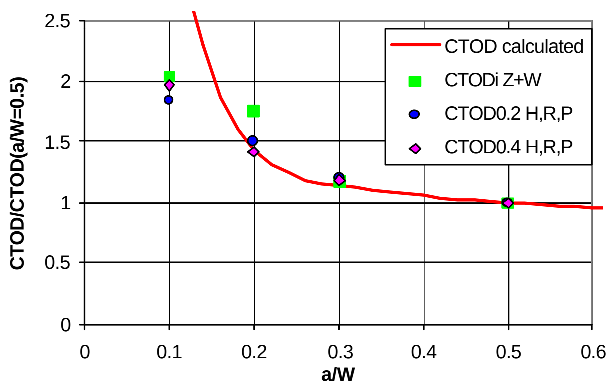

Fig. 7: Comparison of d_{it} predicted values with experimental d_{it} (Z+W from [18], H,R,P from [9]).

5.2 Estimation of plane strain fracture toughness from sub-sided specimens

Eq. (18) and (19) can also be used to correct for differences in OPC, for example to correct fracture toughness data which were measured at a specimen thickness that did not meet the requirement of standards such as [14]. For the extreme cases, plane stress and plane stain, the parameters m and γ are as follows: $m=\sigma_f/R_p$, and $\gamma_{mT}=(\sigma_f/R_p)^2$ for the former, $m=1.48\cdot\sigma_f/R_p$ and $\gamma_{mT}\cong 3.0\cdot\sigma_f/R_p$ for the latter (in case of SSY and a von-Mises material). The effect of the T-stress can be neglected. Therewith, for example the ratio of plane stress to plane strain J_{it}-values is obtained by (19). E.g., for a material with Z=0.7 and σ_f/R_p=1.2 the ratio given by (19) is 2.62.

In general, a sub-sized test specimen of thickness B is neither pure plane stress nor pure plane strain. A plane strain estimate J_{Ic} can be obtained from the apparent toughness, $J_{Ic/a}$, as follows: A lower bound J_{Ic}, denoted by $J_{Ic/LB}$, is given by the size requirement of the testing standard (e.g. [14]) to be

$$J_{Ic/LB} = B\cdot s_f\,/\,25 \qquad (21)$$

If we conservatively assume, that a complete loss of constraint (from plane strain to plane stress) occurs for J> $J_{Ic/LB}$, the additional amount of J, J- $J_{Ic/LB}$, scales according to (19). For the γ_{mT} and m-values given above, this leads to

$$J_{Ic} = J_{Ic/LB} + (J_{Ic/a} - J_{Ic/LB})\cdot\frac{1.48\cdot\left[exp\left(\frac{Z}{3\cdot(1-Z)}\right)-1\right]}{exp\left(\frac{R_p\cdot Z}{s_f\cdot(1-Z)}\right)-1} \qquad (22)$$

Analogously, estimates of plane strain cleavage J_c can be made from cleavage data of sub-sized specimens. In this case, the denominator 25 should be replaced by 200 to obtain a lower bound $J_{c/LB}$, and (13) instead of (19) should be used to scale J-$J_{c/LB}$. Some preliminary comparisons with experimental data from [11, 23, 24] indicate, that the predictions made by (22) are reasonably conservative.

6. Discussion and Conclusions

Being part of the local stressfield, there is no easy way of characterizing crack-tip constraints and of accounting for them in a fracture analysis. Simplifications like the ones suggested in the present paper are always trade-offs with accuracy. The presented approach is an attempt to reduce the required effort of calculations to an engineering level. The assumption that the non-dimensional stress peak γ is a linear combination of the parameters m and T seems to be reasonable. Besides its relative simplicity, the main advantage of the proposed γ_{mT} is its possibility predict effects of IPC as well as OPC. However, further comparisons with experimental and theoretical data are necessary to evaluate the accuracy and consistency of the presented formulas. The factors c_m and c_β, which are chosen here just on the basis of some crude theoretical considerations of SSY, might be optimized by calculating the maximum stress for various geometrical systems and fitting them to the assumption (9).

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