Some Steps Towards Automation of the Crack Compliance Method to Measure Residual Stress Distributions

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Abstract

The crack compliance (CC-) method is an experimental tool to determine the depth profile of residual stresses as well as the stress intensity factor (SIF) due to them, which in a safety analysis often are as important as the stresses. The method is based on cutting the considered part and measuring the resulting strain change at a single suitable point. From this signal, the original residual stress can be calculated by theoretical means. This paper deals with some practical aspects of the CC- method. Some theoretical and experimental modifications are introduced such that the application of the CC-method is simplified and well suited to be partially automated. The required cut is introduced continuously, and the resulting strain change is recorded and stored directly in a spread sheet as a continuous strain vs. time diagram. Therefrom the strain vs. cut depth diagram, which serves as the basis to derive the SIF as a function of cut depth or crack length, is readily obtained. The corresponding mathematical relations, the so-called influence function, are presented in a suitable closed form for some common component shapes, which enable SIFs due to residual stresses to be determined simply by means of a spread sheet or even a hand calculator. Finally, a calculation procedure is suggested that enables the residual stress distribution from the SIF distribution to be obtained nearly automatically, i.e. nearly without any engineering judgement or special expertise of the user.

1 Nomenclature

a	cut length [mm]
E	Young's Modulus [MPa]
h	weight function
K_I	stress intensity factor [N/mm$^{3/2}$]
L	specimen length [mm]
s	distance from cut (Fig.1) [mm]
W	specimen width [mm]
Z	Influence function [m$^{-3/2}$]
ε	strain [m/m]
σ	stress [MPa]

Subscipts:

ref	reference load case
M	measurement point
0	initial
rs	due to residual stress

2 Introduction

To assess the effect of residual stresses on the mechanical behaviour of a structural part it is important to know not only their magnitude at or near the surface, but also their distribution below the surface and across the section. A suitable and straightforward method to determine such stress profiles is the crack compliance method (CC-method), which was proposed by Cheng and Finnie [1 - 2] (see [2] for further references). Basically the same approach was also used in [3] and [4].

The basic idea of the CC-method is to introduce by cutting a narrow slit into the considered body along the section where the residual stresses are to be measured, thereby releasing them at the slit faces. This causes a redistribution of the residual stress field within the entire body. The strain change at any arbitrary location at the surface of the body due to cutting, which obviously contains information about the released stresses, can be measured by a strain gage at a suitable measurement point. There is a unique relation between the released stresses and the strain at the measurement point, which allows the former to be determined from the latter. The corresponding inverse problem is solved in [1 - 3] by a decomposition of the unknown stress field into a number of suitable candidate functions, which are superimposed such that the measured strains are matched.

A somewhat more general procedure to obtain residual stress fields by the CC-method is presented in [5 - 8]. Its key steps are, first, determining the stress-intensity factor (SIF) as a function of cut depth, which follows readily from the strain vs. cut depth curve, and second, inverting this relation to obtain the residual stress by a step-by-step inversion technique. The main advantages of this procedure, which is summarised in the present paper, are the following: It does not require any pre-assumptions or engineering judgements regarding the expected stress-field, and there are practically no convergence problems to deal with. Therefore this method has the potential to become a simple to use "black box" - tool. In the present paper these features of the CC-method are emphasised. It is shown that its application to obtain residual stresses in plates of various shapes is very easy and straightforward. An important subject are the influence functions, i.e. the required relations between the measured strains and the corresponding SIF. Some new solutions for various rectangular plates and beams are given in closed form approximations, which are well suited to be used for data processing in a simple spread sheet.

3 Principle Relations

As mentioned in the introduction, a cut has to be introduced along the plane where the residual stress are to be measured, as shown in Fig. 1, as an example, for the case of a rectangular plate with a cut along its central section. In an overall view, the cut can be regarded as a perfect crack, so the basic equations of linear elastic fracture mechanics can be used to establish the required mathematical relation between the residual stress and the strain change at the measurement point. It can be easily shown [5 - 8] that the SIF at the tip of the above mentioned cut due to the residual stress (for the sake of simplicity we restrict ourselves to Mode-I-cases and to normal stresses with respect to the cut plane) is given by

$$K_{Irs}(a) = \frac{E'}{Z(a)} \frac{d\varepsilon_M}{da} \qquad (1)$$

where ε_M denotes the strain measured during the cutting process as a function of the cut depth a at the (arbitrary) point M (in the example shown in Fig. 1 two measurement points M1 and M2 are considered; one at y=0 at the rear surface, the other in a distance s from the cut at the front surface), and E' the generalised Young's modulus (i.e. E'=E for plane stress and $E'=E/(1-\nu^2)$ for plane stain). Z(a), called the "influence function", is a unique function that depends on the component geometry, on the cut plane and on the location of the measurement point M, but not on the residual stress distribution. Determination of Z(a), which characterises the sensitivity of the measurement point M with respect to the stress release at the cut plane, is the crucial and - considering the theoretical and computational effort - the most demanding step of the CC-method. However, Z(a) needs to be determined only once for a certain geometry and measurement point. In section 4, solutions of Z(a) for some common shapes and measurement points are given.

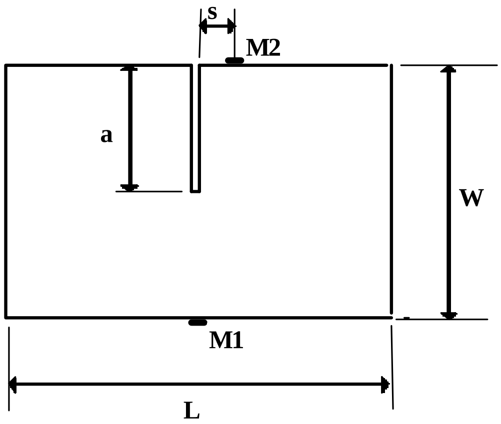


Fig. 1: Rectangular plate, cut along its centre plane; strain measurement points M1 and M2

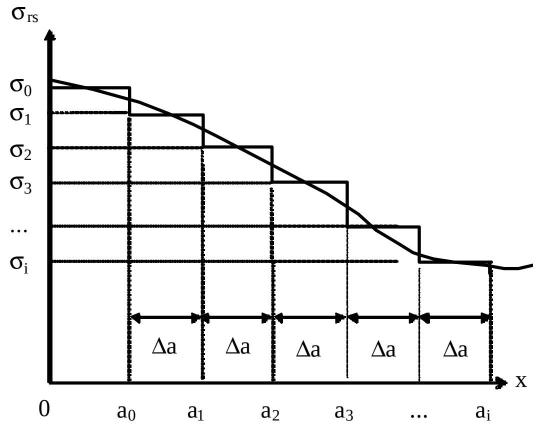


Fig. 2: Approximation of the stress distribution by incremental steps

$K_{Irs}(a)$ results from the normal residual stresses acting prior to cutting, $\sigma_{rs}(x)$ (the x axis being chosen such that it coincides with the crack line, or cut plane, respectively), by the general relation

$$K_{Irs}(a) = \int_0^a h(x,a) \cdot \sigma_{rs}(x) \cdot dx \qquad (2)$$

where h(x,a) denotes the so-called weight function, which is universal for a given crack geometry and available for many systems [9]. In principle, $\sigma_{rs}(x)$ can be obtained by inversion of eq. (2). A suitable technique for this task is given in section 5.

4 Influence Functions

Being a unique function independent on the stress distribution, Z(a) as defined in (2) can be calculated by considering a certain, arbitrary load case, i.e. by

$$Z(a) = \frac{E'}{K_{Iref}(a)} \cdot \frac{de_{Mref}}{da}(a) \qquad (3)$$

where $K_{Iref}(a)$ and $\varepsilon_{Mref}(a)$ denote the stress intensity factor and the strain at M, respectively, for the considered loading case [7]. In the following, solutions are given for the class of systems shown in Fig. 1, which means for beams or rectangular plates with strain measurements at the rear or the front surface. The solutions are presented as simple closed form formulas, which are suitable to be used for data-processing by means of a spread sheet They are obtained by dimensional analysis and curve fitting based on the numerical results presented in [7].

a) Beam; strain measurement at the rear surface (M1 ; L>2W, cf. Fig. 1):

- for a/W≤0.2:

$$Z(a) = \frac{-2.532}{(W-a)^{3/2}} \sqrt{1 - 25 \cdot \left(a/W - 0.2\right)^2} \cdot \left[5.926 \cdot \left(0.2 - a/W\right)^2 - 0.288 \cdot (0.2 - a/W) + 1\right] \qquad (4a)$$

- for 0.2<a/W<1:

$$Z(a)=\frac{-2.532}{(W-a)^{3/2}} \tag{4b}$$

b) Rectangular plate; strain measurement at the rear surface (M1 cf. Fig. 1; 1<L/W<2;)

- for a/W≤1-0.4L/W:

$$Z(a)=\frac{-2.532}{(W-a)^{3/2}}\sqrt{1-\frac{\left(a/W-1+0.4\,L/W\right)^2}{(1-0.4\,L/W)^2}}\cdot\left[\begin{array}{l}1+\left(-3.268+4.597\cdot L/W\right)\left(1-0.4\,L/W-a/W\right)^2-\\ \left(-0.028+0.316\,L/W\right)\left(1-0.4\,L/W-a/W\right)\end{array}\right] \tag{5a}$$

- for 1-0.4L/W<a/W<1:

$$Z(a)=\frac{-2.532}{(W-a)^{3/2}} \tag{5b}$$

c) Beam or rectangular plate, strain measurement at the front surface (M2, cf. Fig. 1)

$$Z(a)=\sum_{i=1}^{6}c_i\cdot\left(\frac{a}{s}\right)^i \tag{6}$$

where the coefficients c_i are given in Table 1. The validity of this polynomial approximation is limited to a/s<2 (for deeper cuts, the sensitivity is in general no longer sufficient anyway).

s/W	c_1	c_2	c_3	c_4	c_5	c_6
0.001	0.2255	-4.9429	9.2692	-6.9913	2.4322	-0.3246
0.01	0.1516	-4.5116	8.3325	-6.1229	2.0609	-0.2649
0.1	0.1639	-4.5030	7.9836	-5.7590	1.9162	-0.2442

Table 1: Coefficients c_i of eq. (6) for a/s<2 and three different measurement positions s or plate widths W, respectively (see Fig. 1).

5 Determination of the Stress Profile

From a known $K_{Irs}(a)$ as obtained from eq.(1), it is possible to calculate the residual stress distribution $\sigma_{rs}(x)$ by inversion of eq. (2). A straightforward way to solve this problem is by the step-by-step procedure described in [10]. The residual stress distribution $\sigma_{rs}(x)$ is approximated by a series of small steps as shown schematically in Fig. 2, so the stress level at each step can be calculated by applying eq. (2) to a hypothetical, incrementally prolonging crack. The average stress of the first increment, σ_0, which represents the average stress acting near the front surface in the range $0<x<a_0$ (where $a_0 \ll W$ should be fulfilled) is obtained from the well known relation between the stress and the SIF of a short edge crack, i.e:

$$\sigma_0=\frac{K_{Irs}(a_0)}{1.12\cdot\sqrt{\pi\cdot a_0}} \tag{7}$$

In order to calculate the average stress level σ_1 of the next step (i.e. the average stress in the range $a_0<x<a_0+\Delta a$), we extend the hypothetical crack by the increment Δa and apply eq. (2) again, and so on for the subsequent steps. Denoting the length of the hypothetical crack after i prolongation increments Δa by a_i (i.e., $a_i=a_0+i\cdot\Delta a$), and the average stress in the corresponding interval $a_{i-1}<x<a_i$ by σ_i (see Fig. 2) eq. (2) is approximated by

$$K_{Irs}(a_i) = s_0 \cdot \int_0^{a_0} h(x,a_i)\cdot dx + \sum_{j=1}^{i-1} s_j \cdot \int_{a_{j-1}}^{a_j} h(x,a_i)\cdot dx + s_i \cdot \int_{a_{i-1}}^{a_i} h(x,a_i)\cdot dx \qquad (8)$$

which allows σ_i to be determined. The resulting step distribution converges to the exact solution $\sigma_{rs}(x)$ as $\Delta a\rightarrow 0$. Some difficulties (mathematical instabilities) may arise in the sensitive region near the rear surface (i.e for W-a<<W), because the used weight function might be inaccurate in this range [11]. There are two ways to overcome this difficulty: either by using a weight function that exhibit the correct limiting behaviour as $W\text{-}a\rightarrow 0$, or changing the calculation procedure as follows: For W-a<<W (which practically means about a>3W/4), (8) can be replaced by

$$K_{Irs}(a_i) = \sum_{j=0}^{i} \left\{ \frac{3.97\sigma_j \cdot \Delta a \cdot \left[0.264W + 0.736a_i - (a_0 + j\cdot\Delta a)\right]}{(W-a_i)^{3/2}} + \frac{1.46\sigma_j \Delta a}{(W-a_i)^{1/2}} \right\} + 2\sigma_i \sqrt{\frac{2}{\pi} q\cdot(W-a_i)} \qquad (9)$$

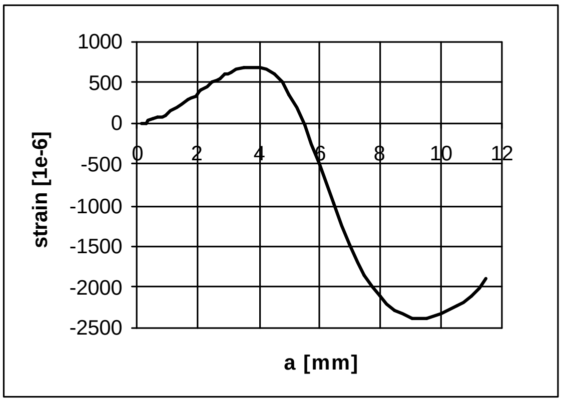
to deliver the values of $\sigma_i(x=a_i)$ [10]. The non-dimensional factor q, which is about 0.03, can be determined from the condition of a continuous transition from eq. (6) to eq. (7) at the chosen transition crack depth, and then kept constant for the further steps.

6 Experimental Procedure and Discussion

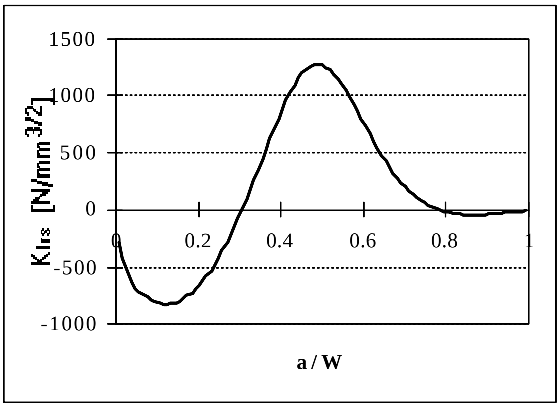
The simplicity and capability of the described method is demonstrated by an example of a residual stress measurement performed on a beam of W=12mm and thickness B=6mm made of a quenched and tempered high strength steel. To introduce residual stresses the beam was pre-bent in 4-point bending to a strain of about 4% at the surface, and then unloaded. The cutting was done at a constant cutting rate $\dot{a}$ by means of continuous EDM (electric discharge machining), which produced a slot width of about 0.3 mm. The strain was measured at the rear surface (M1 according to Fig. 1) by a strain gage. Its signal as well as the actual cut length were recorded as a function of time and directly transferred to a spread sheet by a Spider-8 data recording equipment. Fig. 3(a) shows the measured strain as a function of the cut depth. Therefrom, by a simple spread sheet calculation program, the stress intensity factor K_{Irs} (Fig. 3(b)) is obtained by eqs. (1) and (4), and the distribution of the residual stress (Fig. 3(c)) by the inversion procedure described in section 5. The obtained residual stress distribution is in good agreement with the one that is expected from theoretical considerations, regarding the magnitudes and the general shape.

Beam-shaped or rectangular specimens as considered in section 4 and in the example above are of rather general use, since most essentially two-dimensional components (plates, tubes, profiles) can be sliced to specimens of rectangular shape. Of course the stress relieve that is due to the corresponding pre-cutting has to be recorded and superimposed to the stress field obtained by the CC-method.

In the same way as described above more complex residual stress distributions can be measured. Even rather odd stress fields of odd and discontinuous stress distributions like the ones in the ligament of sharply notched plates [8] or due to surface treatments have been successfully determined.



(a)



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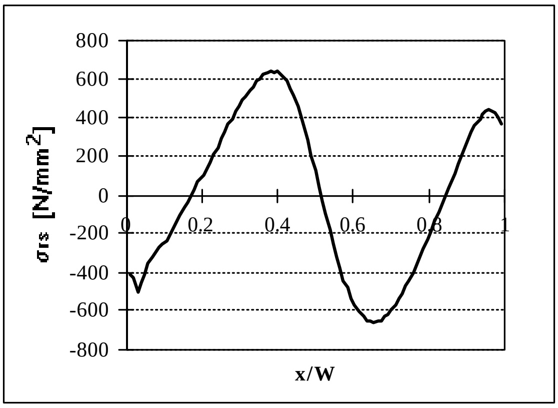

(c)

Fig. 3: Example of residual stress determination in a steel beam with W=12mm

(a) measured strain at M2 (see Fig. 1) as a function of cut depth
(b) stress intensity factor calculated from Fig. 3(a) by means of eqs. (1) and (4)
(c) residual stress calculated from Fig. 3(b) by means of eq. (7), (8) and (9), respectively

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